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Modes of Knowledge Accumulation, Entry Regimes and Patterns of Industrial Evolution

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Modes of Knowledge Accumulation, Entry Regimes and Patterns of Industrial Evolution*

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Abstract

In this work we explore the interplay between entry, selection and innovative learning as determinants of industrial evolution. We propose a model aimed to capture the essential features of learning and competition as drivers of the dynamics. Using both analytical and numerical techniques, we are able to disentangle possible generic properties which robustly hold for a wide range of parameterization. In particular, we identify different generic "evolutionary archetypes" in turn defined by characteristic interactions between entry/exit regimes, learning and industrial structures.

1 Introduction

This work build on the general conjecture, well in tune with evolutionary analyses of economic change, that the primary determinants of industrial dynamics ought to be searched into the underlying process of knowledge accumulation, on the one hand, and market competition amongst heterogenous firms, on the other.

On the ground of a model that formalizes some basic features of technological learning and competitive selection, we shall explore, *first*, the possible existence of *evolutionary invariances*, that is of generic properties of the process of industrial evolution which hold robustly across different learning modes and for a wide range of parameters. Conversely,

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second, we address the properties of industrial structures and change which happen to depend on specific modes of innovative exploration, entry and market selection- i.e. on what elsewhere have been empirically identified and formalized as different *technological* and *markets regimes* (cf. Dosi et al. (1995), Winter (1984),Malerba and Orsenigo (1995),Winter et al. (2000)).

In particular, in the following we shall compare the emergent properties of regimes characterized by: **(a)** different innovative opportunities, **(b)** different degrees of cumulativeness in the probability of innovative success by incumbents; **(c)** dynamic entry barriers, or conversely, learning advantage of potential entrants, and, finally,**(d)** different rates of entry.

The following model is derived from one of those presented in Winter et al. (2000), but, together with other changes, it is extended to cover wider variations in learning and entry patterns. Moreover, here we shall undertake extended Monte Carlo simulations exploring the impact of different learning regimes and of within-regime parameterizations upon a few properties of industrial structure and dynamics. Proxies of the latter include productivity and output growth, patterns of (net) entry and exit, size and age distributions, and measures of concentration, which we all study conditional on diverse regimes.

In Sec. 2 we briefly map the background of this work with respect to both theory and available evidence. The model is presented in Sec. 3. Sec. 4 studies some analytical properties while in Sec. 5 we report on a wide numerical study of the model conducted with Monte Carlo techniques.

2 Industrial Structures and Dynamics: Some Background on Evidence and Theories

The empirical counterpart of this work regards the observed properties of industries persistently characterized by (i) technological learning by incumbents or entrants or both, (ii) entry of new firms, and, (iii) competitive processes at least partly weeding out the heterogeneity in firms populations. Needless to say, these conditions do apply to the overwhelming majority of contemporary industries.

One reviews at greater detail elsewhere (Dosi et al., 1995) the related 'stylized facts' (see also the Special Issues of the *International Journal of Industrial Organization*, 4, 1995 and forthcoming, 2001, and of *Industrial and Corporate Change*, 1, 1997). Here, let us just flag out a few major empirical regularities central to analysis that follows. *First*, industries typically display skewed size distribution which turn out to be relatively stable over time, notwithstanding systematic underlying dynamics in market shares, births and deaths. *Second*, one tends to find - both cross-sectionally and longitudinally - rather robust correlation between (gross) entry and exit rates, even after controlling for industry-specific characteristics, such as scale-related entry barriers, etc. *Third*, there is no apparent impact of profitability conditions upon entry rates. (More on this highly controversial issue in Geroski (1994, 1995) and Dosi and Lovallo (1997)). *Fourth*, the modal fate of entrants is grim (Geroski, 1995) but, possibly, a few outliers turn out to be major drivers of long-term growth of productivity and output (one of the most favorable discussions of the evidence on the point is in Baldwin (1995)). *Fifth*, jointly with the foregoing cross-industry 'stylized facts', one observes equally robust inter-industry differences in the values of statistics such as concentration, firms age distributions, and together in the characteristics and distributions of innovators.

How does one account for all that? Telegraphically, let us recall the major interpretations addressing at least a subset of the foregoing empirical patterns.

Given space limitations, one cannot engage here in any in-depth controversy with that enormous IO literature attempting to rationalize whatever observation by building ex-post corresponding Nash equilibria, often through some quite imaginative reconstructions of history, under the sole constraint of preserving the notion of forward looking micro rationality and collective intertemporally consistent equilibria. In this respect we generally agree with Sutton's point that "explaining everything ex-post" largely stands for "explaining nothing" (Sutton, 1998). However one cannot avoid placing our contribution vis-à-vis a somewhat germane literature which one could call of "rational evolution". Paradigmatic examples of the genre include Jovanovic (1982) and Ericson and Pakes (1995). Empirical regularities that this perspective addresses - at least qualitatively - regard primarily asymmetric size distributions and different growth/death rates conditional on age (Jovanovic, 1982), and collective invariances in industrial structures notwithstanding persistent dynamics on relative, micro, competitive positions (cf. Ericson and Pakes (1995) and related works). In a nutshell, both streams of analysis of "rational evolution" share some acknowledgment of micro diversity and also of the paramount role of market interactions in shaping the destiny of individual firms. However, they both confine the interpretation of the evidence just to modeling exercises that warrant also the consistency between micro rationality and collective outcomes. Indeed, they do that, in our view, at the price of violating a few "stylized facts", such as those regarding quite "inertial" asymmetries in performances, "irrational" entry process and market interactions selecting over seemingly "disequilibrium" micro features. Together, as Kaniovski (2001) argues, the consistency between micro behaviors and collective outcomes is obtained at the price of some mathematically corner-cutting assumptions which basically rule out most aggregation and strategic interdependence issues.

We must admit that it is hard for us to subscribe such interpretations of industrial dynamics fundamentally driven by "rational" explorations and equilibrium market tests: for example we find hard to swallow idiosyncratic competencies of individual firms (a very robust hypothesis, indeed) coupled with unbiased, collectively shared, expectations on their very means and variances (on the contrary a rather bizarre idea); or microtechniques entailing an infinite number of infinitesimally small firms whose aggregation nonetheless yields finite (and common-knowledge) output quantities; or commonly shared "technological expectations" linking search efforts and outcomes[1].

The "bounds" approach, pioneered by Sutton (1991, 1998), is much nearer and, indeed, is significantly overlapping with the perspective wherein the model below finds its roots.

In an extreme synthesis, both share the epistemological commitment to finding relatively robust, empirically falsifiable, predictions on the characteristics of industrial structures - orthogonal to the nuances of fine tunings of individual corporate behaviors, but powerfully influenced by relatively inertial properties of industry-specific technologies and demand patterns. Moreover, both Sutton's and "evolutionary" views on industrial structures and dynamics are meant to yield also cross-sectional predictions - addressing questions like "why is sector A more concentrated than sector B?" etc. - in addition to cross- industrial invariances - such as the already mentioned generic occurrence of skewed size distributions[2].

The "bound approach" and the "evolutionary approach" (in the interpretation explored here), however, depart on some underlying conjectures regarding the determinants of both "bounds" and adjustment processes. Sutton's perspectives suggests that technological and demand-related factors make bounds on industrial structures effective via some no-arbitrage conditions - entailing corresponding Nash equilibria on industry-specific entry processes. Moreover, so far, the "bounds approach" has almost entirely

[1]Of course, no theory should be ask renounce courageous abstractions and simplifications: it is the very essence of theorizing itself. Our point is, however, that this style of interpretation displays - paraphrasing from another context Colin Camerer- a dramatically low ratio of evidence to theory (Camerer , 1995). Moreover it does not appear to display much robustness to the relaxation of the most far-fetched hypotheses (for example, what happens if one allows out-of-equilibrium adjustments?).

[2]Different "rational evolution" models of course are likely to yield different predictions on micro dynamics, as Pakes and Ericson (1998) show. However the models of this family, so far, have not shown much interest, with the exception of the latter, in exercises of mapping between "types of industries" and "types of industrial dynamics".

neglected learning processes. (But, we conjecture, developments in this directions might uncover also further areas of overlapping with evolutionary interpretations.)

Conversely, the evolutionary perspective theoretically gambles on further departures from either "rational" microfoundations or collective equilibrium outcomes. Rather, it grounds its empirical predictions concerning industrial structures and dynamics upon the identification of underlying regularities, first, in *microeconomic learning processes* - of different kinds, concerning in principle technologies, organizational forms, behavioral patterns -, and second, in *interaction mechanisms* - entailing also specific selection processes within heterogeneous populations of firms.

Incidentally, note that this latter perspective involve a rather radical subversion of the strategy-centered perspective of most current IO in that it identified crucial explanatory factors of industrial organizations in the features of learning processes which are, to some extent, specific to particular bodies of knowledge and which shape the modes and rates at which collections of firms, in each production activity access notional opportunities of innovation and imitation. Indeed, it is a theme of inquiry which has found Dick Nelson as a pioneer (among many works, cf. Nelson (1981), an insightful assessment of the evidence available twenty years ago that in the economic community one just begins to appreciate...).

There are at least two fundamental theoretical issues here related to the foregoing *destrategising conjecture*. In its weak form, the conjecture suggests that some basic properties of industrial structures and industrial change may be easily understood as quite generic consequences of quite a few processes of experimentation and imperfect trial-aderror learning - without invoking any more sophisticated form of "strategic rationality" on the part of individual agents. The strong form of the same proposition suggests that also intersectoral differences in structures and processes of change might be understood, as a first approximation, independently from detail micro strategies, but just with reference to intersectoral differences in the process of technological accumulation[3].

Evolutionary theories of industrial change have indeed made significant progress in the understanding of why industrial structures are what they are and also, of what explains taxonomic differences across them. Just to provide an extremely concise map, confined to modeling exercises, major roots rest in Winter (1971) and Nelson and Winter (1982) and subsequent developments include Iwai (1984a,b), Winter (1984), Metcalfe

[3]So, as an extreme illustration, consider, say, IBM which became for a long period the dominant player in the international computer industry, certainly owing also to its systematically successful strategies. However, the foregoing conjecture suggests that, even without IBM as such, the computer industry would have followed rather similar dynamics, fundamentally shaped by the nature of collectively shared learning processes. Indeed, the results from "history-friendly models" put forward in Malerba et al. (1999) corroborate the hypothesis.

(1992), Kwasnicki (1996), Silverberg and Verspagen (1994),Mazzucato (2000), Dosi et al. (1995), Winter et al. (1997, 2000), Malerba et al. (1999)). Within such expanding literature, however, one might possibly distinguish some sort of "first generation", whereby the primary task has been to show the consistency of the exercise and the empirical plausibility of the results - a fundamental initial task indeed[4]. A more recent "second generation", however, attempts to derive also finer empirical predictions and it does so within two diverse but quite complementary styles of analysis.

First, "history-friendly models" add richer, history-based, phenomenological details to the formal representation of specific industries (cf. Malerba et al. (1999)). Together, one must of course demand an account for a much wider set of empirical phenomena. At the other, complementary, extreme, one begins to explore rather parsimonious "reduced form" models, which however ought to be able to generate a few generic collective statistics on industry structures and dynamics, even when on purpose neglecting most historical, industry-specific, characteristics. This is what one has begun to do - together with other scholars from different quarters, in Winter et al. (1997, 2000) -, and this is also the spirit of the model below. Hence, in the following, we neglect on purpose history-specific features of technological learning and market competition but try to identify, first, possible invariances in the revealed outcomes, and, together, second, possible transitions across "evolutionary archetypes", dependent on some threshold values in the rates of microeconomic learning, rates of entry of new firms, entrants size and death rules.

3 The Model

The model studied here is a close descendent of Winter et al. (1997, 2000), which in turn have close ancestors in Nelson and Winter (1982) and Winter (1984). Let us first provide a qualitative overview.

In the "benchmark case" below we model an industry with a number of firms variable throughout its history - that number being determined by stochastic entry at each "period" and competition-driven market selection. Competition, well in tune with Nelson and Winter (1982) affects differential growth rates via investment opportunities, stemming in turn from different current gross profits. Firms which shrink below a certain minimal size "die". Entry rates, in probability, depends on the number of incumbents. Learning alike one of the models studied in Winter et al. (2000) concerns only the productivity of physical capital (and, as labor inputs are assumed to be constant, we set them

[4]A good deal of the formal explorations in Nelson and Winter (1982) clearly focus on this target and implicitly so do a lot of subsequent works in the formal evolutionary tradition.

to zero, without loss of generality). In terms of empirical plausibility, the assumption of solely capital-related learning is indeed a rather awkward one. However, in order to construct some relatively simple "thought experiment" on learning, selection and growth, this is the most congenial candidate. As one finds in Winter et al. (2000), learning on the labor productivity dimension - indeed a more realistic assumption - generally implies that the long-term dynamics depend also on the shape of demand function. But this introduces a further interaction term into a study wherein we would like to focus, as a first approximation, on the supply side within a sort of "partial disequilibrium analysis"[5].

In the benchmark case below, both incumbents and entrants are able to stochastically learn (In that, the benchmark resemble what many of us have come to conventionally call a "Schumpeter II" industrial regime: cf. Malerba and Orsenigo (1995), Dosi et al. (1995), amongst others).

The model consider an industry I whose firms i are characterized at each time by a productivity $\pi_i(t)$ and a capital $k_i(t)$ evolving under the learning dynamics and the competitive pressure, respectively. The initial conditions of the industry are given by $N(0)$ firms whose initial capital $k_i(0)$ is randomly drawn from an uniform distribution with support $(\epsilon, \epsilon + M_k)$. The initial (log) productivity $\log(\pi_i(0))$ is randomly drawn from a Gaussian distribution of mean 0 and variance 1. At each subsequent time step t the following actions are performed:

1. Exit The firms whose capital $k_i(t)$ is smaller that a given fraction ϵ of the whole industry capital leave the industry.

2. Production and price determination Each firm assign its total physical capital to production contributing to the total supply with $q_i(t) = \pi_i(t)k_i(t)$. We assume that the output price is related via a given "demand function" to the aggregate supply. Specifically, if $Q(t) = \sum_i q_i(t)$ we set:

$$p(t) = H(Q(t)) \tag{1}$$

where H is in general a non-increasing function of its arguments. In order to not introduce any specific hypothesis on the nature of the produced good and its characterization on the demand side we assume a demand function of the form $H(Q) = 1/Q$. In so doing we keep the total industry revenue (and gross profits) invariant, irrespectively of actual output.

3. Capital update If $p(t)$ is the industry-wide price of output, the (gross) profit of firm i reads:

$$p(t)\,\pi_i(t)\,k_i(t) \tag{2}$$

[5] A significant step forward, which we are beginning to explore, involves indeed learning on both labor and "machines" efficiencies.

Assuming that the fraction of profit reinvested in production, λ, is constant over time and identical across firms, the capital of firm i is updated according to

$$k_i(t+1) = k_i(t)\,(1-d) \;+\; \lambda k_i(t)\,p(t)\,\pi_i(t) \tag{3}$$

where d is the capital depreciation rate (assumed homogeneous and constant) and where the expression of output in term of capital has been substituted.

4. Entry. The number of entrants $n_{\text{in}}(t)$ is obtained from a Poisson distribution with mean proportional to the number of incumbents firms present in the industry $< n_{\text{new}} >= r\,N(t)$. Their capital is randomly drawn from a uniform distribution ranging from $\epsilon\,K(t)$ to $(\epsilon + M_k)\,K(t)$, and their productivity is drawn from a lognormal distribution whose mean and variance are the actual weighted mean and variance of the logarithms of the incumbents productivities[6].

5. Learning. Each firm present in the industry (including the just entered ones) has a probability 1/2 of drawing a productivity increase $\log(\pi_i(t+1)) = \log(\pi_i(t)) + \epsilon_i$. The extent of the increase ϵ_i is randomly generated from an exponential distribution with mean α (and hence variance $\alpha^2/2$).

In particular below we shall study the effect on industry structure and dynamics of the following parameters

- ϵ the minimal allowed capital share, corresponding to the death threshold
- M_k the maximal initial capital share of entrants
- α the mean of the exponential distribution from which productivity increments are drawn
- r the ratio of the average number of the entrants cohort to the industry population

These parameters, together with the parameters d and λ defined in (3) and kept constant, completely specify the model.

4 Some analytical properties of the model

In order to better understand some basic properties of the model, before tackling its fullfledged version, let us begin by studying some analytical properties of two special cases.

[6]This entry rule is conceived in order to simulate a "smooth" insertion of new firms in the productivity distribution of the incumbents.

The closed industry without learning

Consider a "closed" industry of N firms, each one endowed from the beginning with a given capital productivity π_i and where no exit and entry dynamics take place. Let us analyze the long-term fate of the industry, in terms of asymptotic values of aggregate quantities, together with the role played by the aggregate demand functions H.

Summing over the firms in (3) one can obtain the evolution of the total capital $K(t) = \sum_{i \in I} k_i(t)$, which reads

$$K(t+1) = (1-d)\, K(t) \;+\; \lambda \qquad . \tag{4}$$

Then, as $t \to \infty$, aggregate capital converges exponentially toward the asymptotic value $\bar{K} = \lambda/d$.

In order to simplify the recursive relations driving the industry dynamics let us rescale individual firms' capitals by introducing the variable $x_i(t) = d/\lambda k_i(t)$ so that (3) becomes

$$x_i(t+1) = (1-d)\, x_i(t) \;+\; d \frac{x_i(t)\, \pi_i(t)}{\sum_j \, x_j(t)\, \pi_j(t)} \tag{5}$$

and the evolution of the "rescaled" aggregate capital[7] $X(t) = \sum_i x_i(t)$ is given by

$$X(t+1) = (1-d)\, X(t) \;+\; d \qquad . \tag{6}$$

The variables x_i remain positive and the stationary points $\bar{x}^* = (x_1^*, \ldots, x_N^*)$ of (3) satisfy the relation

$$\sum_i \pi_j x_j^* = \pi_i \quad \forall i \in N \tag{7}$$

i.e. they are the vertices of the N-dimensional simplex[8].

In order to identify the asymptotic state of the industry it is necessary to identify "stable" equilibria amongst those defined by (7). For this purpose, let us consider the industry aggregate productivity defined as the average firms productivity weighted with their capital shares:

$$\Pi(t) = \sum_i \; \pi_i(t)\, \frac{x_i(t)}{X(t)} \qquad . \tag{8}$$

Using (5) and (6), it is straightforward to write the evolution equation for Π which reads

$$\Delta\Pi(t) = \Pi(t+1) - \Pi(t) = \frac{d}{\Pi(t)\,((1-d)X(t)+d)} \left(\sum_i \; \pi_i^2(t)\, \frac{x_i(t)}{X(t)} - \Pi(t)^2 \right) \qquad . \tag{9}$$

[7]Notice that the rescaling of capital makes the parameter λ to disappear. This parameter, when identical across firms, has indeed the sole purpose to set the scale of the aggregate capital and can be conveniently eliminated.

[8]Eq. (3) is basically identical to a discrete time replicator dynamics. In the canonical definition of the latter (Weibull, 1998) the payoff of each "type" does depend only on the shares of each "type", while in (3) it depends also on the total population $X(t)$. This difference, however, turns out to have a negligible effect for the present analysis.

The right hand side of (5) is positive for any t and the aggregate productivity Π is a Lyapunov function for the dynamic defined by (5). We may thus conclude that for any initial condition[9] $(k_i(0), \ldots, k_N(0))$ *the closed industry evolves toward a finite asymptotic aggregate capital and the share of this capital possessed by the most productive firm is asymptotically* 1.

Moreover the last factor on the right hand side of (5) is the variance of the productivity over the industry so that the rate of growth of average productivity is proportional to the "heterogeneity" of the productivity distribution over the firms. Due to the strong similarity of (5) with a replicator dynamics, it is not surprising to see the Fisher Law (Metcalfe, 1998) at work here.

Finally, it is easy to show that with a demand function of the form $H(Q) = 1/Q^{\alpha_p}$, the previous statement concerning the asymptotic dynamics of the industry is true irrespectively of the value of α_p. This means that for what concerns the qualitative behavior of the industry, the exact specification of the demand function is irrelevant[10].

The closed industry with learning

Next, let us show that the introduction of some generic random growth dynamics for firm productivities, i.e. "learning", in the previous closed industry model does not substantially change the industry long term features.

Suppose that the productivity of each firms increases over time following a stochastic growth of the form $\ln(\pi_i(t+1)) = \ln(\pi_i(t)) + \epsilon_i(t)$, where $\epsilon_i(t)$ is a random variable with density function $q(\epsilon)$ and support in $[0, +\infty)$, equal for each firm.

Let $p_t(k, \pi)$ the probability density to find at time t a firm with capital k and productivity π. Such a process on productivity growth together with (3) are enough to define an evolution equation for this density

$$p_{t+1}(k, \pi) = \int_0^{+\infty} dk' \int_0^{+\infty} d\pi' p_t(k', \pi') q(\pi - \pi') \delta\left(k - k'(1 - d + \lambda \frac{\pi'}{Q(t)})\right) \tag{10}$$

where $\delta(x)$ is Dirac delta function.

From this expression it is possible to derive the asymptotic behavior of the industry total output

$$\bar{Q}(t) \sim \sqrt{\lambda \bar{K}}\ e^{\sigma_\epsilon t} \tag{11}$$

where σ_ϵ is the variance of the productivity growth [11] (for the detailed derivation see

[9] Excluding of course the zero-measure set formed by the non-dominant simplex vertices.

[10] When $\alpha_p \neq 1$ however the limit value of the aggregate capital is a function of the best productivity in the industry.

[11] The exponential growth of $Q(t)$ is the effect of a multiplicative process on the firms learning. If one would model the productivity growth as an additive process, the ensuing growth in total production would be linear.

Appendix A).

Moreover, one can also extract the leading term (in the asymptotic expansion for high t) of the shares distribution. At time t it reads

$$p_t(k) = \frac{1}{(1-d)^t}\, p_0(\frac{k}{(1-d)^t}) \tag{12}$$

where $p_0(t)$ is the initial shares distribution.

A notable property of (12) is that it shows how the competitive dynamics defined in (4) progressively "shrink" the capital shares distribution, scaling down all its moments. Notice also that the density function in (12) does not conserve the total capital. This is perfectly consistent with our previous findings: when $t \to +\infty$ the amount of capital possessed by any finite measure set (i.e. by a given fraction of the firms population) goes to 0 while the total capital concentrates in the hands of a single firm (which constitute a zero measure set in the continuous description used here).

This "closed industry"with learning basically boils down to the same asymptotic properties, in term of industry structure, as the non-learning case. The progressive "shrinking" of the size distribution implies that any "exit" condition imposed on the industry in term of an exit threshold on firm's capital, leads, irrespectively of its value, to the formation of a monopoly [12].

4.1 Asymptotic values for some aggregate quantities

Next, let us analyze our complete benchmark model where, together with competitive and learning dynamics, we insert an entry and exit condition for all firms, "opening" the industry to an external flux of capital.

We start by studying the number of firms presents in the industry. Due to the stochastic nature of the model we are interested in a mean value:

$$N^* = \lim_{T \to +\infty} \frac{1}{T} \sum_{\tau=0}^{T} N(\tau) \tag{13}$$

First of all let us note that this value cannot diverge. This comes from the very nature of the entry and exit rules defined above: when new firms enter the industry, they not only increase the number of previously present firms, but also contribute a positive increment to the aggregate capital. This implies that the number of firms that are likely to exit the next time step are incremented from these entry events more than proportionally. To be a bit more rigorous, let $p_t(k)$ be the probability density of finding a firm with capital k at time t (with $k > \epsilon K(t)$) and $N(t)$ the number of present firms.

[12] Without exit conditions, the identity of the monopolist possessing the total mass of the shares distribution is of course not asymptotically fixed

The average number of entrants will be $N_{\text{in}} = rN(t)$ and the average inflow of capital $K_{\text{in}} = rN(t)K(t)M_k/2$. The rise in total capital produced by entry will shift the lower bound on "surviving" capital and possibly induces both incumbent and just-entered firms to leave the industry. On average, the number of exiting firms can be approximated with

$$\begin{aligned} N_{\text{out}} &= N(t) \int_{\epsilon K(t)}^{\epsilon(K(t)+K_{\text{in}})} dk'\, p(k') + \\ & \quad rN(t)\Phi\left(rN(t)\frac{\epsilon}{M_k}(\epsilon + \frac{M_k}{2})\right) \end{aligned} \tag{14}$$

where the first term is the contribution to exit coming from the incumbents firms distribution and the second is the contribution of the just-entered firms, which are uniformly distributed between $\epsilon K(t)$ and $(\epsilon + M_k)K(t)$. To simplify the expression we use a function $\Phi(x)$ defined according to:

$$\Phi(x) = \begin{cases} x\,, & 0 < x < 1 \\ 1\,, & x > 1 \end{cases} \tag{15}$$

The expression in (14) increases at least quadratically (for N small). So, it suggests that when N is small enough, entry does not immediately affect the successive exit flow but for higher N, the entry of new firms will be followed by a higher exit effect, thus producing a total net decrease in the total number of firms.

We can use (14) to extract an upper bound on $N(t)$. Suppose that $N(t)$ happens to be so high that the second term in the right hand side of (14) equals $rN(t)$. This would imply that the whole set of entrants will be pushed out from the industry by the same capital they brought in, leaving invariant the total number of firms in the industry: there is no way to increase the total number of firms above this level. Hence we can obtain an upper bound for the number of firms equating the second term in the right hand side of (14) to $rN(t)$. It reads

$$N_{\text{u.b.}} = \frac{M_k}{\epsilon\,(M_k + \epsilon)\; r} \tag{16}$$

This of course represents a gross overestimation of the actual average number of firms. Nevertheless, its existence implies that for any finite value of the entry and exit parameters the number of firms is limited.

It must be stressed, however, that the foregoing analysis closely apply only to high enough values of the entry coefficient r. When entry rates are low, we have to consider the actual evolution of incumbents in the industry i.e. one has to modify the previous equation taking into account the actual "shrinking" in firms distribution described in (12).

The fact that we can define an average number of firms N^* allows us to obtain an estimate for the asymptotic average value of the total industry capital. The idea is to modify (4) to accommodate for entry and exit dynamics. Taking into account the average values for the entrants initial capital and the lower bound for exit flow, equation (4) can be written as:

$$K(t+1) = (1-d)K(t) + \lambda + N_{\text{in}}(\epsilon + \frac{M_k}{2})K(t) - N_{\text{out}}\epsilon K(t) \quad . \tag{17}$$

But (on average) $N_{\text{in}} = N_{\text{out}} = rN^*$ so that the asymptotic total capital computed from (17) reads

$$K^* = \frac{\lambda}{1-d+0.5\,r\,N^*\,M_k} \quad . \tag{18}$$

This expression clearly suggest the existence of two phases[13]: when $r\,N^*\,M_k < 2\,d$, that is for small enough values of the entry ratio r and initial capital M_k, the industry is in a "finite" phase and the aggregate capital asymptotically converges toward a finite value as in the "closed" case — the latter being however a lower bound with a value $K^* = \lambda/d$. On the other hand, when $r\,N^*\,M_k > 2\,d$ the industry is in a "divergent" phase and its total capital diverges exponentially[14] when $t \to +\infty$.

The approximation in (18) is however numerically unreliable for two reasons. First, the capital outflow due to exit is overestimated, since the capital shares possessed by exiting firms will be in general less than the threshold ϵ. Second, the capital inflow is overestimated, as discussed above, since the net firm entry ratio can be significantly less than r[15]. This notwithstanding, the expression in (18) provide a good approximation of the numerical results below if one allows for a single parameter γ capturing the whole "impactedness" of entry. In this spirit we have fitted the expression:

$$K^*_{\text{fit}} = \frac{\lambda}{1-d+0.5\,r\,N^*\,M_k\,\gamma} \tag{19}$$

as a function of γ on our simulations results (described below). With an estimated $\gamma = .7131$ the standard deviation of the simulation results from the value predicted by (19) is 0.0192 (and the maximal deviation is 3.9%) despite the fact that the values involve a span of 3 orders of magnitude[16].

Finally, for what concerns the behavior of the total output, in the "finite" phase it is described by an expression alike (11) with α in place of σ_ϵ, while in the "exploding"

[13]Here and throughout "phase" stands for the collection of the characteristic properties of system dynamics which turn out to be qualitatively invariant within regions of the parameters space. Hence such a notion does not have any bearing in terms of evolutionary "stages" over time.

[14]Notice that even if the only parameters that explicitly appear in (18) are those pertaining to entry rules, the other parameters may influence the asymptotic values through the value of N^*.

[15]Here it is the net firms entry ratio and not the gross one (i.e. r) that matters because only firms surviving a full productive cycle can contribute to the next period aggregate capital.

[16]This estimate is performed considering only the value associated to a finite limit in (19).

phase this expression has to be corrected to take in account the exponential increase of the aggregate capital.

5 Numerical Analysis

Let us now present a wider exploration of the benchmark model described above, based on Monte Carlo simulations for different parameterizations

The evolving structure of the industry is captured by various aggregate observables. Together with the total number of firms and total aggregate capital, already discussed, we consider the average growth rate of the industry productivity, defined as a weighted mean of the firms productivity $\pi_{\mathrm{I}}(t) = \sum_i \pi_i(t) k_i(t)/K(t)$, and, as a measure of industry concentration, the "rescaled" entropy defined as

$$S_I(t) = -\frac{1}{\log(N(t))} \sum_{i \in I} \frac{q_i(t)}{Q(t)} \log(\frac{q_i(t)}{Q(t)}) \tag{20}$$

whose value ranges from 0 for a monopoly to 1 when shares are equal among firms. Moreover we consider both the average age of firms and the average age of firms weighted with their market share[17].

The previous analysis shown how the entry and exit parameters affect both the number of firms and the aggregate capital of the industry, leading to the existence of two different "phases". It is thus natural to begin our numerical exploration with the analysis of how entry and exit parameters affect other relevant aspects of the industry behavior. As we shall see the conclusions above, based on rough approximations of the entry/exit dynamics will be confirmed, at least qualitatively, by numerical simulation.

Let us start by considering the simulation results[18] shown in Fig. 1 for different values of the rate of entry r and the maximal initial share M_k keeping all other parameters fixed[19]. The total number of firms increases with the rate of entry but the effect is much less pronounced at high entry rates. In this region in fact the dynamics described by (14) dominates and a large part of just-entered firm leave immediately the industry, so that the "net" effect of entry reach a maximum. The effect of the maximal entry size also follows the prediction of (14): when it decreases the total number of firms increases until it values is smaller that ϵ; then its effect tend to disappear.

[17]The possible difference between these statistics allow us to discern, for each parameters region, if firms who have more market share are the younger or the older ones.

[18]In this plot and in the following ones the points shown are the result of averaging over 50 independent simulations

[19]In what follows when not otherwise specified the parameters used in simulation get the default values given in Tab. 1

Analogous conclusions can be drawn studying simulations for different values of the rate of entry r and of the exit bound ϵ, shown in Fig. 2. We observe again a "saturation effect" on the entry rate while, as expected, the number of firms increases when the exit bound is lowered.

Since the behavior of aggregate capital is accurately captured by (18) as a function of the average number of firms we do not plot it but refer instead to the analytical expression above.

We turn next to the study of quantities for which no prediction has been made via analytical approximations. One of the most interesting variables is the rate of growth of the average productivity shown in Fig. 3 and Fig. 4. Consider its behavior together with that of entropy (which is, to repeat, an inverse measure of concentration) shown in Fig. 7 and Fig. 8 and with measures of average age (Fig. 5 and Fig. 6)

Note that we are comparing here the outcomes of different entry and exit regimes, *conditional on the same notional opportunities*, as expressed by the probability distribution on productivity increments, and the same ability of accessing it by incumbents and entrants. As one can see, when the entry rate is high but both the maximal initial size and the exit threshold are low, the industry is characterized by a high value of productivity growth and by small concentration, and is composed by relatively young firms. Notice also that in this *fast growing* region the aggregate capital remains finite.

If, however, the maximal entry capital is increased, the industry enter the *divergent* phase, characterized by an unbounded aggregate capital, wherein the high turbulence is disrupting the productivity growth process. Here, the massive entry of firms with relative high amount of capital leads to the continuous expulsion of big shares of the incumbents population, so disturbing the underlying learning possibilities.

On the other hand, if the entry rate is lowered and the entry threshold increased, the industry shows a more "sedate" behavior, eventually reaching a *strong oligopolistic* structure in which entry is a rare event and just medium and big firms survive. Old incumbents, even if their productivity is almost stationary, easily survive since they are protected by their relatively big size.

To summarize, we have identified three main "phases" describing the behavior of our benchmark model. First, a *divergent phase* is characterized by a high turbulence and slow productivity increase. Moreover the *finite phase* can be splitted in two part, namely a *fast growing* phase and a *strong oligopolistic* phase characterized by a nearly stagnant industry.

From a qualitative point of view, the results highlight the fundamental role as driver of the dynamics of "reasonable" rates of (small) entry suitable to undertake multiple innovative searches without however being "massive" enough as to destroy the process of adaptation and of differential growth of heterogeneous competing entities.

Next let us study the behavior of the model with varying degrees of opportunity which firms face concerning their potential productivity growth.

5.1 Changing relative opportunities for incumbent firms

Here we are interested in the effect on the industry aggregate behavior produced by a modification of the parameter α setting the scale of the distribution from which productivity growth is drawn.

In this spirit we perform a comparison, for the range of the entry/selection parameters analyzed above, of different Monte Carlo simulations obtained varying the opportunity parameter. While its default value was 0.01, here we consider both values equal to 0.04 and 0.08.

A first obvious effect on the average productivity rate of growth, is that the latter, as expected, scales with α. Less intuitively, we find that the effect of the parameter α over the industry behavior varies depending on the "phases" characterizing the industry.

The learning dynamic has indeed a negligible effect in the "divergent" and "oligopolistic" phases. As can be seen from Fig. 9 and Fig. 10, if the entry rate and the initial share are both very high or both very low, the average age and the entropy computed remain almost invariant to changes in the opportunity parameters.

For intermediate situations, we observe that when the degree of opportunity grows, the total number of firms decrease, and the concentration increases. So, in general, more opportunities for both the incumbents and the entrants correspond to a more selective market, producing as a net effect a slight advantage for older firms. Average age confirm this argument: this statistics increase, even if weakly, as degree of opportunity grows[20].

5.2 The benchmark versus a "Schumpeter I" learning regime

In order to analyze the asymptotic properties of a regime of the "Schumpeter I" genre, we obviously have to depart from benchmark model and consider an industry in which incumbents cannot improve their productivity, so that they produce throughout their life with the technique extracted at birth.

An intuitive way of obtaining a model like this starting from the benchmark one presented above would be to simply "turn off" the learning process of incumbents leaving

[20] The same conclusion is confirmed studying the ratio between average age and average weighted age, not shown due to space constraints

all the other features of the model unchanged. Note, however, that this procedure does not work. If one does that, indeed, the productivity of any firms converges toward the same asymptotic value and the variance of the productivity distribution goes to zero. In fact, in each iteration the less productive firms are likely to disappear while above average productivity firms conquest market shares. Therefore, the selective mechanisms progressively reduces heterogeneity in productivity and in absence of a learning mechanism which constantly reintroduce diversity, the productivities of all the firms present in the industry tend toward the same value. Putting it another way, no faithful distribution of "clones" from the incumbents environments appears to be able to fuel any persistent evolutionary drive.

A richer account of a "Shumpeter I" regime is a model in which entrants have an innovative advantage in probability, while incumbents learning remains turned off: this is done by giving a positive drift to the distribution from which entrants extract their initial productivities, with respect to the revealed productivity distribution of incumbents[21].

As can be seen from Fig. 11, the behavior of this model is very similar to the benchmark's behavior for high values of the entry rate r. However, when this parameter is lowered, no "oligopolistic phase" appears. The absence of such a phase is easily understood: since the relative productivity of different incumbent firms is not subject to random fluctuations, there is no advantage in having a bigger size, and even few and small entrants are able to displace the industry core firms. This same features are reflected also in the degrees of concentration and in average age. The entropy for this model is always near to 1, implying a constantly uniform distribution of market shares, while the industry average age remains lower than in the benchmark case, where the formation of the "oligopolistic core" produces an increase in the average lifespan of incumbency.

Allowing for a drift in the entrants productivity, does also change the system behavior in the "divergent" phase. As can be seen in Fig. 12 the rate of productivity growth is less affected by an increase in initial sizes and entry rates than in the benchmark case, since now there is no learning process by the industry "core" which could be disturbed by an increase (and widening) of industry turbulence.

It is also interesting to analyze what happens if one introduces a *negative* drift in entrants productivity, so that their mean productivity is less then the industry average[22]. (Notice that a dynamic in which the drift gets a negative value is more in line with what actually found in empirical investigations).

[21] In order to perform an analysis comparable with previous findings we assign to the drift the same value of the "opportunity" parameter α in the benchmark model (see Tab. 1).

[22] We model this regime setting the drift value at the same level of the positive drift case, with a reversed sign.

First of all, the negative drift provides again an implicit advantage to incumbent firms and thus restores the existence of an "oligopolistic phase". When the rate of entry is low, a relatively small number of incumbents has a low probability to be perturbed by the arrival of new firms, since there is a low probability that at least one amongst the latter will posses a higher productivity than the incumbents. As can be seen in Fig. 13 the number of firms in the negative-drift model is very similar to the benchmark.

An interesting effect however emerges in relation to the "divergent" phase. When the turbulence of the industry is very high (with a high rate of entry) and wide (with large initial capitals), affecting also the industry core, the smaller incumbents are likely to be pushed out from the industry by the "impactedness" of new firms, with bigger capital even if with smaller productivity. As shown in Fig. 14 the net effect is the steady *decrease* of the industry aggregate productivity.

5.3 Size distribution

On purpose, so far, in order to succinctly describe different learning, entry and selection regimes we have entirely neglected a few statistics describing distributions of characteristics across the population of firms, regarding e.g. size, productivity, age, growth, ecc.

Here, due to space limitations, let us limit ourselves to the analysis of the quantity most ubiquitously studied, namely the distribution of (the log of) firm sizes.

The structure shown by the share distribution of our model is rather robust and largely rests upon the variables characterizing the entry and exit dynamics[23].

First of all it must be noted than a stationary share distribution emerges (asymptotically) both in the "finite" and in the "divergent" phases discussed above. The existence of a stationary distribution is in contrast with the closed models analyzed in Sec. 4 and comes from the "openness" of the system, i.e. from the constant flux of firms and capital in and out of the industry[24]

Due to the exit condition, the share distribution has support $[\epsilon, +\infty)$. Its shape can be divided in two regions according to the entry dynamics. In the region $[\epsilon, M_k]$ there is a constant influx of firms and their number is relatively high. Firms in this region are more likely to exit and are directly competing with the entering ones. This

[23]Instrumental to the identification of the general structure of the share distribution has been the development of a "diffusive" continuous-time approximation of our model. For more details see Bottazzi (2001)

[24]Trying a comparison with "physical" systems, we could say that the industries described in Sec. 4 are "closed dissipative systems" converging toward the minimal entropy state, the monopoly, while the model with entry and exit is an "open system" which evolve toward a stationary out-of-equilibrium state

region constitutes the "fringe" of the industry, the locus of higher turbulence. On the other hand, the region $[M_k, +\infty)$ is constituted of firms which have, so to speak, won so far the "competitive struggle" by reaching higher market shares. This is the core of the industry. Here turbulence is lower and the firms in this region are more clearly ranked. Only on a longer term, they are possibly displaced from their current position and eventually reduced to the fringe by more productive competitors that steadily, but relatively slowly, "climb" the successive positions of the rank ladder.

Due to their different nature, in these two regions the distribution of the (log) shares shows a very different shape. In fact, as can be seen in Fig. 15, it turns out than in the $[\epsilon, M_k]$ region the distribution is rather flat while it decreases linearly in $[M_k, +\infty)$. A rescaling of both the *market impactedness*, M_k, and of the *selection coarseness*, ϵ, has the sole effect of translating the distribution. If however one increase the first keeping constant the latter, giving an higher weight to the entry dynamics, the "flat" region is increased and consequently the industry "core" is reduced. Above some threshold value, the turbulence invades all the industry and the divergent phase is entered.

The effect of the entry rate can be analogously interpreted. When r is high, entry has a greater impact and one should expect a higher concentration of firms in the "fringe" region. When it is low, the "core" region is dominant and one expects longer, fatter tails in the distribution. An example of the effect obtained varying the entry rate can be seen in Fig. 16.

6 Conclusions and outlook

This work, we hope, has added some novel insight into the understanding of the interplay between learning, entry and selection regimes in determining industrial structure and dynamics.

In that respect we were able to identify three quite generic "evolutionary archetypes" or "phases" (in the sense spelled out in Sec. 4) in the evolution process, in *primis* dependent on the rates and impactedness of entry, namely, *first*, an "oligopolistic phase" where neither (very low) entry rates nor learning processes are able to shake the industry out of a quasi-stationary state; *second*, at the opposite extreme, a "divergent phase", whereby massive entry is disruptive of any process of adaptation and self-organization of the industry, and *third*, inbetween what we could be tempted to call the "healthy evolutionary phase" whereby entry is sufficient to guarantee multiple search trials by a few entrants but also a progressive competitive adaptation of the most successful incumbents. Interestingly, it is primarily in this phase that the rates and modes of learning by incumbents and entrants do influence the structure of the industry and its changes.

More specifically, regarding productivity growth, a remarkable property of our model is that long-term rates are obviously influenced by the level of notional opportunities, but also, holding opportunities constant, by entry regimes. Our results robustly suggest that the most conducive regime is one where a relatively numerous population of (small) entrants steadily coexist with a "core" of industrial incumbents. That distinction bears its implications also in terms of size distribution, generally displaying the coexistence of a "fringe" region and a core region characterized by Pareto-type size (market shares) distributions, well in accordance with the empirical evidence. The distinction core/fringe matches also our earlier distinction of "oligopolistic" and "divergent" phases of evolution so that only the core appears in the former and only the fringe in the latter.

More generally, our investigation of some fundamental invariances in evolutionary processes in the industrial domain has led us to disentangle some seemingly robust mechanisms of interaction between the competitive dynamics, on the one hand, and learning processes, on the other.

The analysis is also amenable to exercises of a more normative flavor, investigating the "efficiency" of different evolutionary regimes. This is a task we are currently beginning to undertake.

APPENDIX

A Closed industry with learning

Consider the evolution equation presented in (10). The associated evolution equation for the total production $Q(t) = \sum_{i \in I} q_i(t)$ can be immediately obtained multiplying both sides for π and k before integrating. It reads

$$\begin{aligned} Q(t+1) &= \int_0^{+\infty} dk \int_0^{+\infty} d\pi \; k \, \pi \, p_{t+1}(k,\pi) = \\ &= m_\epsilon \left((1-d)\frac{K(t)}{N} + \lambda \right) \; + \; (1-d)Q(t) \; + \; \lambda \frac{M_{1,2}[p_t]}{Q(t)} \end{aligned} \tag{21}$$

where $m_\epsilon = \int dx \; x \; q(x)$ is the average productivity growth and where we have denoted the higher moments of the p density as

$$M_{i,j}[p_t] = \int_0^{+\infty} dk \int_0^{+\infty} d\pi \; k^i \, \pi^j \, p_t(k,\pi) \tag{22}$$

Since the dynamics of $k(t)$ is characterized by (4), the previous expression shows that $Q(t)$ oscillate around an equilibrium value $\bar{Q}(t)$ which grows exponentially over time. Indeed the last term in the right hand side of (21) provides the leading contribution and under the approximation of independence between k and π (i.e. supposing $p(k,\pi) = p_k(k)p_\pi(\pi)$), one obtains

$$\bar{Q}(t) \sim \sqrt{\lambda M_{1,2}[p_t]} \tag{23}$$

which after substituting the definition in (22) gives the expression reported in (11).

In order to extract the leading term of $p_t(k,\pi)$ as $t \to +\infty$ it is useful to consider the double Laplace transform (the characteristic function) of the density, which is given by

$$\tilde{p}_t(l,m) = \int_0^{+\infty} dk \int_0^{+\infty} d\pi \; e^{-k\,l} \; e^{-\pi\,m} p_t(k,\pi) \tag{24}$$

This function is defined and analytical in the positive real part complex half-plane. Applying the Laplace transform to both sides of (10) one obtains after some algebra:

$$\tilde{p}_{t+1}(l,m) = \tilde{q}(m) \; \int_0^{+\infty} dk' \int_0^{+\infty} d\pi' \; e^{-k'\,(1-d)\,l} \; e^{-\pi'\,m} \; e^{-\frac{\lambda k' \pi'}{Q(t)}\,l} \; p_t(k',\pi') \tag{25}$$

From (23), the coefficient of the mixed exponent in k' and π' is going to zero as $t \to +\infty$ and in order to extract the leading term it can be put equal to 1. With this approximation the integration can be formally performed obtaining:

$$\tilde{p}_{t+1}(l,m) = \tilde{q}(m) \; \tilde{p}_t((1-d)\,l,m) \tag{26}$$

which can be immediately solved for the t-th step density

$$\tilde{p}_t(l,m) = \tilde{q}(m)^t \tilde{p}_0((1-d)^t\,l,m) \tag{27}$$

Without any lack of generality we can assume that the initial assignment of productivity is independent from the initial assignment of capital shares, and choose as initial distribution of productivities the same distribution q so that $p_0(k,\pi) = p_0(k)\, q(\pi)$. The solution obtained anti-transforming (27) reads

$$p_t(k,\pi) = \int_{c_1} dl \int_{c_2} dm\, \tilde{p}_0((1-d)^t\, k)\, \tilde{q}(m)^{t+1} = \frac{1}{(1-d)^t}\, p_0(\frac{k}{(1-d)^t})\, q^{\star(t+1)}(\pi) \quad (28)$$

where the $\star$ denote the convolution operator and c_1 c_2 are suitable contours of integration in the complex plane.

References

Baldwin, J.R., (1995) The Dynamic of Industrial Competition, Cambridge, Cambridge University Press.

Bottazzi, G. (2001) "Continuous-time limit of a modified WKD model of industry evolution", LEM Working Paper, S. Anna School of Advanced Studies, Italy.

Camerer, C. (1995) "Individual decision making" in Kagel, J.H. and Roth, A.E. (Eds.), The Handbook of Experimental Economics, Princeton, N.J., Princeton University Press.

Dosi, G., Marsili, O., Orsenigo, L., Salvatore, R. (1995) "Learning, market Selection and evolution of industrial structure", Small. Bus. Econ. 7 pp. 411-436

Dosi, G., and Lovallo, S., (1997) "Rational Entrepreneurs or Optimistic Martyrs? Some Considerations on Technological Regimes, Corporate Entries, and the Evolutionary Role of Decision Biases" in Garud, R., Nayyar, P., Shapira, Z. (eds) Technological Innovation: Oversights and Foresights, Cambridge University Press, pp. 41-68

Ericson, R. and Pakes, A., (1995) "Markov-perfect industry dynamics: a framework for empirical work" Rev. Econ. Stud. 62 pp. 53-82

Geroski, P.A., (1994) Market Structure, Corporate Performance and Innovative Activity, Oxford University Press: Oxford.

Geroski, P.A., (1995) "What do we Know about Entry?", International Journal of Industrial Organization, 13 pp.421-440.

Iwai, K., (1984a) "Shumpeterian Dynamics I: Technological progress, Form growth and Economic Selection" J. of Econ. Behavior and Organization, 5 pp. 159-190

Iwai, K., (1984b) “Shumpeterian Dynamics Part II: Technological progress, Form growth and Economic Selection” J. of Econ. Behavior and Organization, 5 pp. 321-351

Jovanovic, B., (1982) ” Selection and Evolution in Industry”, Econometrica, 50 pp.649-670.

Kaniovski, Y.M., (2001) “On a rational expectation equilibrium”, Univ. of Bolzano, Italy, Working Paper.

Kwasnicki, W. (1996) “Innovation Regimes, Entry and market Structure”, J. of Evol. Econ., 6 pp. 375-409

Malerba, F., and Orsenigo, L., (1995) ”Schumpeterian Patterns of Innovation”, Cambridge Journal of Economics, 19 pp.47-65.

Malerba, F., Nelson, R., Orsenigo, L. and Winter, S., (1999) ”’History Friendly Models of Industry Evolution: The Computer Industry.” Industrial and Corporate Change 8

Mazzucato, M. (2000) Firm Size, Innovation and Market Structure: The Emergence of Market Concentration and Instability, Aldershot, UK and Northampton, MA, USA, Edward Elgar

Metcalfe, S., (1992) “Variety, Structure and Change: An Evolutionary Perspective on the Competitive Process”, Revue d’Economie Industrielle pp. 46-62

Metcalfe, S., (1998) Evolutionary Economics and Creative Destruction, London, Routledge

Nelson, R. R., (1982) “Research on Productivity Growth and Productivity Differences: Dead Ends and New Departures”, J. of Econ. Lit. XIX pp. 1029-1064

Nelson, R. and Winter S., (1982) An Evolutionary Theory of Economic Change, Cambridge, Mass. The Belknap Press of Harvard University Press

Pakes, A., and Ericson, R. (1998) “Empirical Implications of Alternative Models of Firm Dynamics” J. of Econ. Th. 79 pp. 1-45

Silverberg, G. and Verspagen, B. (1994) “Learning, Innovation and Economic Growth: A Long Run Model of Industrial Evolution”, Industrial and Corporate Change, 3 pp. 199-223

Sutton, J. (1991) Sunk Costs and Market Structure, MIT Press: Cambridge, Mass.

Sutton, J. (1998) Technology and Market Structures, MIT Press: Cambridge, Mass.

Weibull, J.W., (1995), Evolutionary Game Theory, Cambridge, Mass., The MIT Press.

Winter, S. (1971) "Satisficing, Selection, and the Innovating Remnant", Quart. J. of Eco. pp. 237-261

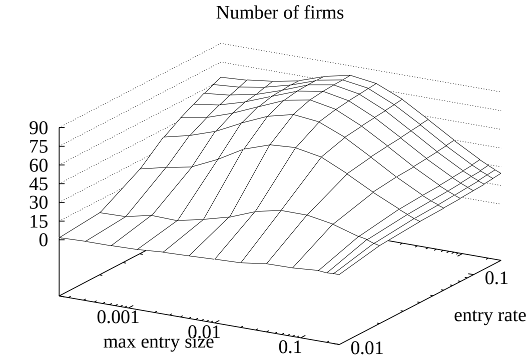
Winter, S. (1984) "Schumpeterian Competition under Alternative Technological Regimes", Journal of Economic Behavior and Organization 5 pp. 287-320

Winter, S.G., Kaniovski, Y.M., and Dosi, G. (1997) "A Baseline Model of Industry Evolution", Interim Report IR-97-013/March, International Institute for Applied Systems Analysis, Laxemburg, Austria.

Winter S., Kaniovski Y.M. and Dosi G. (2000) "Modeling industrial dynamics with innovative entrants" Structural Change and Economics Dynamics 11 pp. 255-293

param.	default	param.	default
ϵ	0.01	r	0.05
M_k	0.1	α	0.01
d	0.3	λ	0.6

Table 1: Default values of the model's parameters used in simulations.



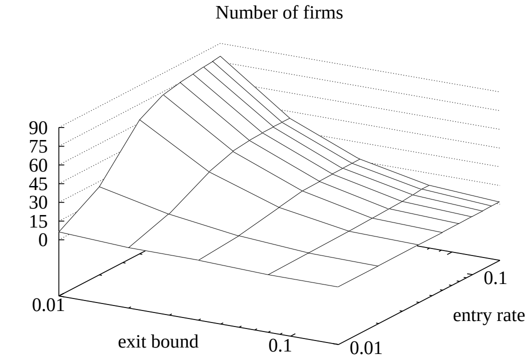
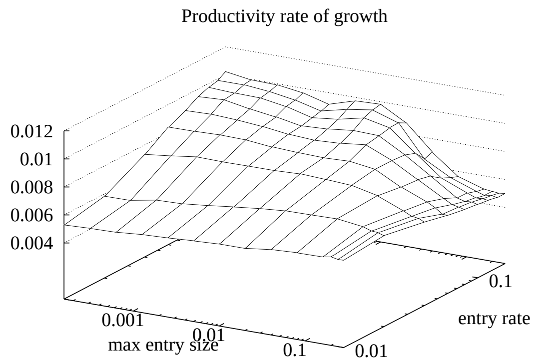


Figure 1: Mean number of firms N^* as a function of the rate of entry r and of the upper bound of initial capital distribution M_k.

Figure 2: Mean number of firms N^* as a function of lower bound on capital shares ϵ and of the rate of entry r. The simulation are performed with $M_k = .04$.



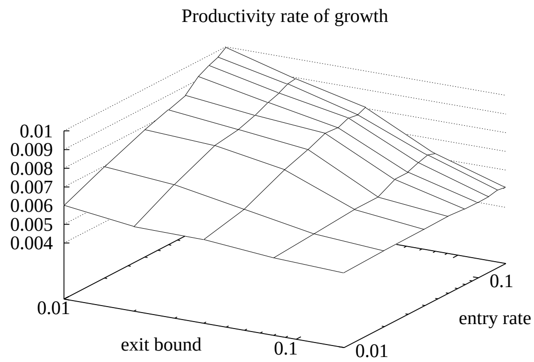


Figure 3: Rate of productivity growth as a function of the rate of entry r and of the upper bound of initial capital distribution M_k.

Figure 4: Rate of productivity growth as a function of lower bound on capital shares ϵ and of the rate of entry r.

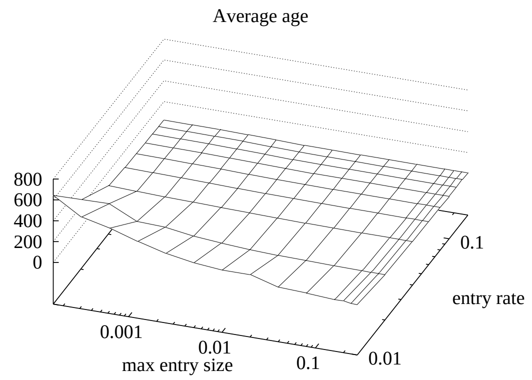


Figure 5: Average firms age as function of the rate of entry r and of the upper bound of initial capital distribution M_k.

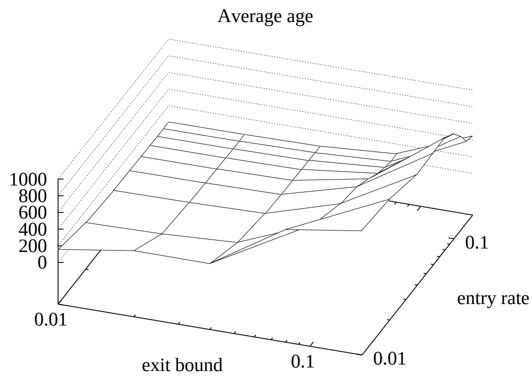


Figure 6: Average firms age as a function of lower bound on capital shares ϵ and of the rate of entry r.

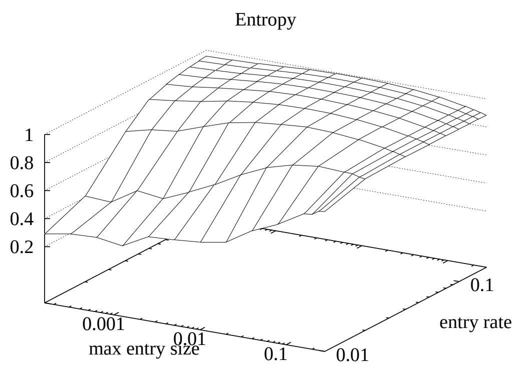


Figure 7: The rescaled entropy as a function of the rate of entry r and of the upper bound of initial capital distribution M_k.

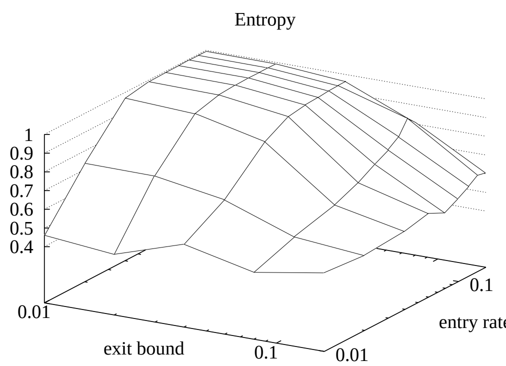


Figure 8: The rescaled entropy as a function of lower bound on capital shares ϵ and of the rate of entry r.

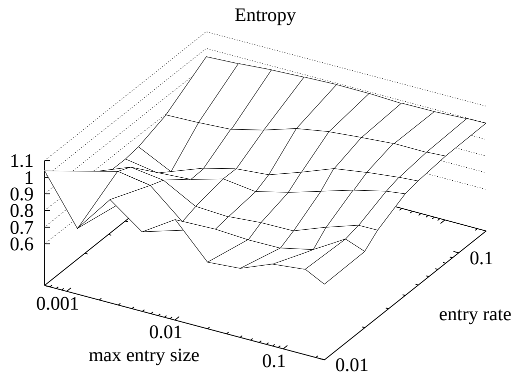


Figure 9: The ratio of entropy values obtained with $\alpha = .2$ to those obtained with $\alpha = .4$, as a function of the rate of entry r and of the upper bound of initial capital distribution M_k.

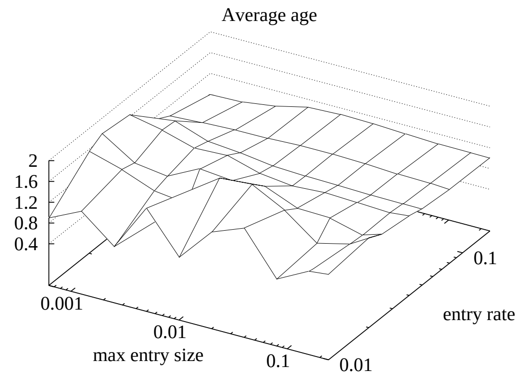


Figure 10: The ratio of the average age obtained with $\alpha = .2$ to that obtained with $\alpha = .4$, as a function of the rate of entry r and of the upper bound of initial capital distribution M_k.

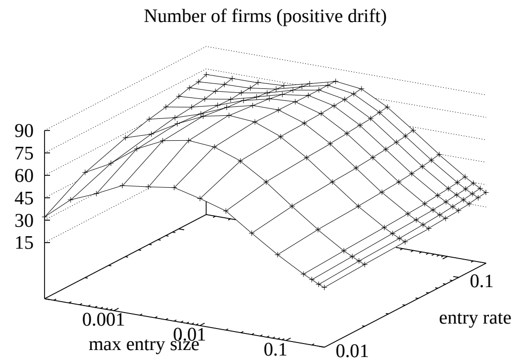


Figure 11: Total number of firms as a function of the rate of entry r and of the upper bound of initial capital distribution M_k for the "Schumpeter I" model with positive drift.

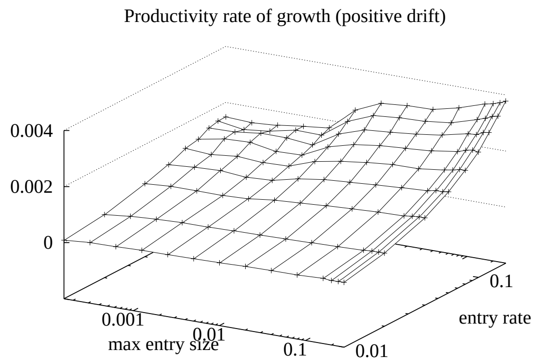


Figure 12: Rate of productivity growth as a function of the rate of entry r and of the upper bound of initial capital distribution M_k for the "Schumpeter I" model with positive drift.

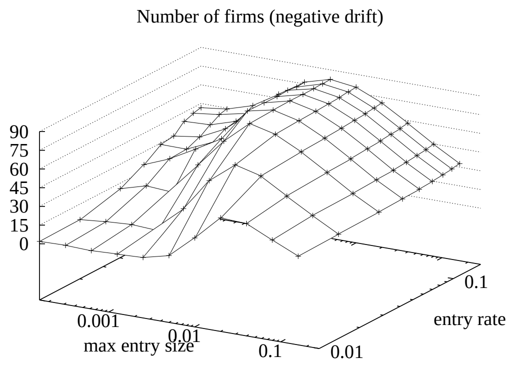


Figure 13: Number of firms for the "Schumpeter I" model with negative drift. The values are plotted as function of the rate of entry r and of the upper bound of initial capital distribution M_k.

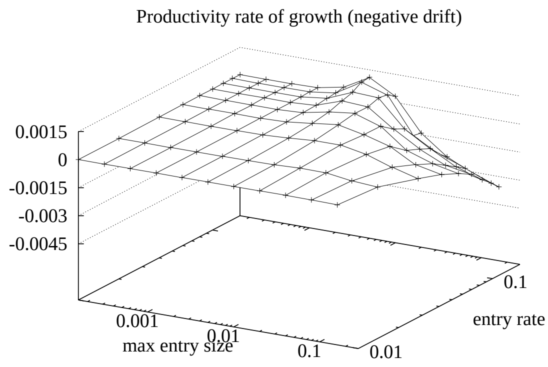


Figure 14: Average rate of productivity growth for the "Schumpeter I" model with negative shift. The "divergent" phase is characterized by a *reduction* in industry productivity.

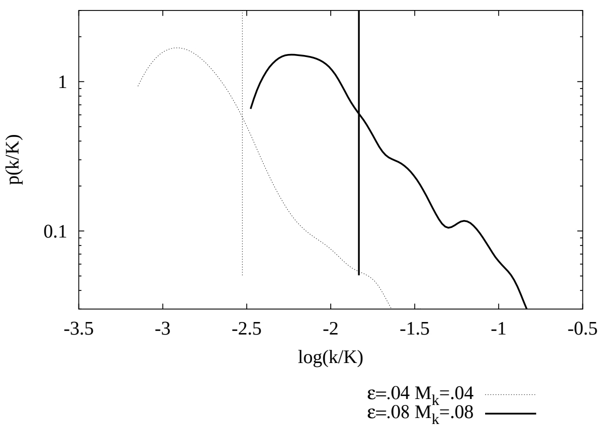


Figure 15: Shares distribution for different parameterization of the entry process. For each curve is also shown (vertical line) the value $\epsilon + M_k$ which provide an estimation of the core lower boundary. Simulations performed with $r = .04$.

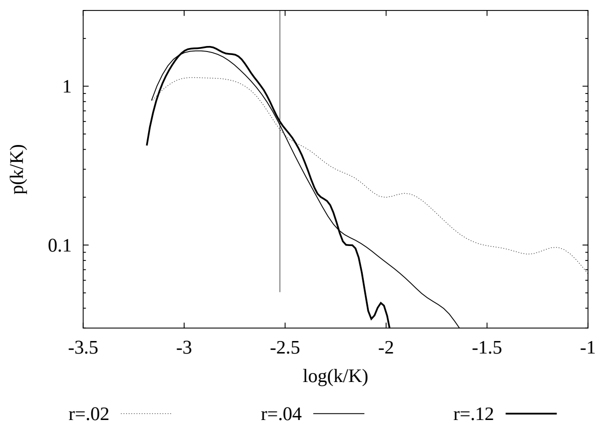


Figure 16: Shares distribution for three different values of the entry parameter r. The simulations are performed with $\epsilon = M_k = .02$. The value $\epsilon + M_k$ is also shown (vertical line).