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2024/14 April 2024
ISSN(ONLINE) 2284-0400

Pricing anomalies in a general equilibrium model with biased learning

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April 27, 2024

Abstract

We investigate the emergence of momentum and reversal anomalies in a general equilibrium model with complete markets and cognitively biased agents, accounting for the presence of representativeness heuristic, conservatism, and anchoring and adjusting in their beliefs. We characterize anomalies by studying return autocorrelation patterns, price gaps following sequences of different events, and relative performances of suitably defined portfolios. These three characterizations are not equivalent. They capture different aspects of mispricing and relate differently to the behavioral heuristics that we consider. Overall, the model is generically able to reproduce the empirical evidence of momentum profits that subsequently revert.

JEL Classification: G41, D53, G12, G14

Keywords: Momentum; Reversal; Biased Learning; Bayesian Learning; Model Misspecification.

1 Introduction

In recent decades, a large body of empirical evidence has accumulated on the existence of common mispricing patterns in different financial markets around the world: returns seem to keep the recent trend for a while (momentum) and then revert toward average levels (reversal). Three groups of studies can be roughly identified to support this evidence. The first group is based on portfolio comparison exercises and has a marked cross-sectional dimension. It revolves around the performance analysis of portfolios composed of securities that performed the best or worst in the recent past. In this spirit, Jegadeesh and Titman (1993, 2001), using US stock data from the 1960s to the 1990s, show that a portfolio long on the best and short on the worst performing stocks over the last 3 to 12 months provides significant positive abnormal returns in the following 3 to 12 months. The abnormal performance disappears on longer horizons. De Bondt and Thaler (1985, 1987), using again US stock data but considering observations from the 1920s to the early 1980s, show that a portfolio long on the worst performing stocks over the last 3 to 5 years (the *Loser* portfolio) significantly outperforms a portfolio long on the best-performing stocks over the same time window (the *Winner* portfolio) over the next 3 to 5 years. Rouwenhorst (1998), Moskowitz and Grinblatt (1999), and Asness et al. (2013) extend the previous investigation, finding evidence of momentum for, respectively, European stock markets, industrial portfolios, and different markets and asset classes. The second group of studies focuses on price movement after earnings announcements. They are based on the idea that stock prices do not adjust immediately after an unexpected earnings announcement, but instead show a predictable drift. Using a sample of US companies from the mid-1970s to the early 1980s, Foster et al. (1984) investigated the cumulative abnormal returns of ten portfolios based on estimated earnings surprises. They found that the earnings surprise is associated in magnitude and sign with the performance of the portfolios in the next 60 trading days. Bernard and Thomas (1989, 1990) confirm the previous findings, considering a larger set of stocks and a longer period of time. Milian (2015) shows that, in recent times, the effect has weakened and reports a form of reversal, especially for liquid US stocks. An extensive critical overview of the literature on post-earning announcement drift can be found in Fink (2021). The third body of literature focuses on the temporal dimension, studying the autocorrelation structure of the time series of individual security returns. Together with the classical contributions of Poterba and Summers (1988) and Cutler et al. (1991), the most notable piece of evidence comes from the analysis of Moskowitz et al. (2012) on different asset classes and different countries during the period 1965-2009. The authors find significant time-series momentum (i.e. positive autocorrelation of rescaled returns) for about one year that partially reverses afterward. Lo and MacKinlay (1990) and Lewellen (2002) combine the cross-sectional dimension with the temporal dimension. They show that a lead-lag effect emerging from the cross-serial correlation of returns contributes to generating portfolio evidence on momentum and reversal.

These mispricing patterns are generally considered inconsistent with standard asset pricing models (e.g. Lucas, 1978). Therefore, several behavioral theories have been proposed to explain them. The idea is that widespread cognitive biases cause a form of underreaction to news that is subsequently corrected, with the delayed correction resulting in a form of overreaction. The *conditional* nature of mispricing, caused by biased and different reactions to sequences of good and bad news, has been used to study anomalies in Barberis et al. (1998). The authors investigate how conservatism and the representativeness heuristic may generate patterns in an infinite-horizon representative agent framework. Bottazzi and Giachini (2022, 2024) extend that analysis to a general equilibrium model with complete markets and generic reinforcing and progressive belief updating rules. Bottazzi et al. (2019), instead, consider an Evolutionary Finance model (Evstigneev et al., 2008, 2016; Bottazzi et al., 2018, 2023) characterized by temporary equilibrium, a complete market, and agents investing in long-lived assets proportionally to expected relative dividends.[1] They show that price anomalies arise from the relative wealth dynamics generated by heterogeneous agents with persistently misspecified beliefs. Other models prefer a *time-series* approach, characterizing price anomalies through the autocorrelation structure and dispensing from explicitly comparing reactions to sequences of good and bad news. For example, Daniel et al. (1998) consider a representative agent characterized by biased self-attribution and overconfidence in a finite-horizon economy. As private and public signals arrive, the returns appear to be positively autocorrelated in the first period and negatively autocorrelated afterward. Hong and Stein (1999), instead, employ a partial equilibrium model in which news watchers interact with momentum traders. They show that as news spreads slowly in the population of news watchers, returns are positively autocorrelated in the first periods and then revert as a consequence of the action of momentum traders. Somehow distancing from the behavioral finance tradition, several efforts have recently been made to explain time-series momentum and reversal in terms of informative asymmetries that affect heterogeneous and (otherwise) rational agents. (see, e.g., Cespa and Vives, 2012; Ottaviani and Sørensen, 2015; Cujean and Hasler, 2017). Luo et al. (2021) mixes the two approaches in a finite-horizon partial equilibrium model with incomplete markets. They show that time-series momentum and reversal can result from the combination of early-informed overconfident investors, late-informed skeptical investors, and a rational risk-averse market maker.

Most of the aforementioned theoretical contributions study the emergence of momentum and reversal in a single-security model under partial equilibrium and incomplete markets. Moreover, they focus exclusively on one definition, either conditional or time series. Conversely, in this paper, we propose a general equilibrium pricing model in which conditional, time-series, and cross-sectional price anomalies can be clearly defined, investigated, and related to agents' behavioral biases. Considering complete markets and homogeneous agents that maximize the

[1] This is equivalent to assuming that each agent decides its portfolio by solving a representative agent model under logarithmic preferences and its subjective beliefs.

geometrically discounted expected utility of consumption over an infinite horizon, we show that the three types of anomalies, although related, are not equivalent. Similarly to Barberis et al. (1998), agents learn by combining two bi-stochastic Markov models, one favoring switching over persistence and one doing the opposite. The weights used to combine the models evolve over time, and their update rule is flexible enough to accommodate *representativeness heuristic*, *conservatism*, and *anchoring and adjusting*.

We derive some general conditions for the emergence of conditional and cross-sectional anomalies. Focusing on two analytically tractable cases, we investigate the emergence of anomalies when different biases are active. We find that conservatism and the representativeness heuristic are not related to specific effects, as the same range of anomalies can be observed or not observed under both biases. In the numerical investigation of the general case, anchoring emerges as a key factor in determining the region of the parameter space where momentum or conditional reversal are observed. At the same time, the degree of conservatism appears to influence the level of agreement between the conditional and time-series definitions, with an increasing effect on momentum and a decreasing effect on reversal. Interestingly, time-series reversal is not much affected by changes in behavioral parameters. The cross-sectional momentum appears generally in line with the predictions of conditional and time-series definitions, whereas the link between cross-sectional, conditional, and time-series reversal appears weaker. This is due to the role that formation and observation periods play in the cross-sectional definition of anomalies. They add a further layer of variability that can lead to a wide range of phenomena. Anchoring still plays a role in favoring momentum or reversal, but the effect appears asymmetric.

The model is generically able to generate short-term momentum followed by long-term reversal, replicating the empirical evidence. However, we do not find evidence that the three types of anomalies, conditional, time-series, and cross-sectional, occur together. For instance, we find evidence of the momentum profits that subsequently revert in cross-sectional terms in regions of the parameter space where conditional reversal does not occur.

2 The model

Consider an Arrow-Debreu economy with N identical agents, a homogeneous consumption good, complete markets, infinite horizon, and discrete time indexed by $t = 0, 1, \ldots$. Call $s_t \in \{0, 1\}$ the state realized at time $t > 0$. The state at time $t = 0$ is $s_0 \in \{0, 1\}$ and is certain. The sequence $\sigma = (s_0, s_1, s_2, \ldots, s_t, \ldots)$ indicates a path and $\sigma_t = (s_0, s_1, s_2, \ldots, s_t)$ is the partial history until time t. The set of all possible paths is Σ, the set of all partial histories until time t is

Σ^t, $\mathcal{C}(\sigma_t) = \{\sigma \in \Sigma | \sigma = (\sigma_t, \ldots)\}$ is the cylinder with base σ_t, and $\mathcal{F}_t$ is the σ-algebra generated by the cylinders $\mathcal{C}(\sigma_t)$. By construction, $(\mathcal{F}_t)_{t=0}^{\infty}$ is a filtration and $\mathcal{F}$ is the σ-algebra generated by the union of filtrations. We indicate with P the true probability measure on $(\Sigma, \mathcal{F})$, such that $(\Sigma, \mathcal{F}, P)$ is a well-defined probability space. We assume that the true data generating process is i.i.d. with $P(s_{t+1} = 1|\sigma_t) = \pi \ \forall t, \sigma$. The expectation is denoted by E and, when there is no subscript or superscript, it is computed with respect to P.

Agents have a constant and homogeneous endowment $e_i(\sigma_t) = e > 0 \ \forall i, t, \sigma$ and share the same subjective measure p on $(\Sigma, \mathcal{F})$. Denoting with $c_i(\sigma_t)$ the consumption of agent i at time t along path σ, the consumption levels solve

$$
\begin{aligned}
\max_{\{c_i(\sigma_t),\, \forall t, \sigma\}} U_i &= \sum_{t=0}^{\infty} \beta^t \, \mathrm{E}_p[u(c_i(\sigma_t)] = \sum_{t=0}^{\infty} \sum_{\sigma_t \in \Sigma^t} \beta^t p(\sigma_t) u(c_i(\sigma_t)) \\
\text{subject to} \quad & \sum_{t=0}^{\infty} \sum_{\sigma_t \in \Sigma^t} q(\sigma_t) \left(e - c_i(\sigma_t)\right) \geq 0 \,,
\end{aligned}
\tag{1}
$$

where $\beta \in (0,1)$ is the inter-temporal utility discount factor, $q(\sigma_t)$ is the price of the Arrow-Debreu security paying 1 if partial history σ_t is realized and zero otherwise, and $u(c)$ is a strictly increasing and strictly concave Bernoulli utility.

We assume that the subjective measure p is absolutely continuous with respect to the true measure P, such that the equilibrium exists and is unique and markets clear in every period. Given the homogeneity of the preferences and endowments of the agents, the consumption of the agent i after a partial history σ_t is $c_i(\sigma_t) = e$, while the price of the Arrow-Debreu security reads $q(\sigma_t) = \beta^t p(\sigma_t)$, $\forall t, \sigma$.

3 Pricing Anomalies

We investigate the dynamics of price using a simple short-lived security. Consider a claim issued on date t with a payoff of one if $s_{t+1} = 1$ and zero otherwise. For an investor holding a long position in this security, the occurrence of the state of nature 1 is "good news" while the occurrence of 0 is "bad news". Given a partial history σ_t, the equilibrium price of this security is $q(1|\sigma_t) = q(\sigma_t, 1)/q(\sigma_t) = \beta p(1|\sigma_t)$, the subjective conditional probability that agents assign to observing state 1 after σ_t multiplied by the discount factor. The return of this security reads $r(\sigma_{t+1}) = (\beta p(1|\sigma_t))^{-1}$ if $s_{t+1} = 1$ and $r(\sigma_{t+1}) = 0$ if $s_{t+1} = 0$.

We study three alternative definitions of momentum and reversal. Since the model is dynamic and encompasses an infinite horizon, we consider the asymptotic values of the quantities of interest to avoid transient effects. The first definition captures the modification of the expected price conditional on the last state of nature realized (Barberis et al., 1998; Bottazzi and Giachini, 2022). Let

$$F_j(\sigma_t) = p(1|\sigma_t, s_{t+1} = \ldots = s_{t+j} = 1) - p(1|\sigma_t, s_{t+1} = \ldots = s_{t+j} = 0)$$

be the rescaled price difference after a sequences of j good news and j bad news.

Definition 3.1 (Conditional momentum and reversal)**.** *Conditional momentum* occurs if $\limsup_{t\to\infty} \mathrm{E}[F_1(\sigma_t)] < 0$, while *conditional reversal* occurs if $\exists j > 1$ such that $\liminf_{t\to\infty} \mathrm{E}[F_j(\sigma_t)] > 0$.

Returns tend to keep the trend if the price of the security after a good state is, on average, lower than after a bad state. In other terms, if prices *underreact* to news. Returns revert if after a sequence of good news the price tends to be higher than after a sequence of bad news, that is, if prices *overreact* to long sequences of concordant news. This definition is the most suited to capture and reproduce evidence on post-earning announcement drift (see, e.g., Bernard and Thomas, 1989, 1990; Fink, 2021).

The second definition focuses on the autocorrelation structure of the returns (Daniel et al., 1998; Hong and Stein, 1999; Ottaviani and Sørensen, 2015). It captures the presence of a form of predictability in the sequences of returns, avoiding any specific conditioning on realized states.

Definition 3.2 (Time-series momentum and reversal)**.** *Time-series momentum* occurs if $\liminf_{t\to\infty} \mathrm{Cov}[r(\sigma_{t+1}), r(\sigma_t)] > 0$, while *time-series reversal* occurs if $\exists j > 1$ such that $\limsup_{t\to\infty} \mathrm{Cov}[r(\sigma_{t+j}), r(\sigma_t)] < 0$.

Momentum occurs if the autocorrelation of returns is positive in the short-run and reversal occurs if the autocorrelation becomes negative in the long-run. This definition is the most suitable for capturing and reproducing evidence on time-series momentum and reversal (see, e.g., Poterba and Summers, 1988; Cutler et al., 1991; Moskowitz et al., 2012).

The third definition, directly inspired by the empirical exercises performed by De Bondt and Thaler (1985, 1987) and Jegadeesh and Titman (1993, 2001), compares the average performance of two portfolios. The first portfolio has a long position of one unit in the risky security defined above and a short position of one unit in a riskless security that pays one unit of consumption good in the following period, whose return is $1/\beta$. The second portfolio has opposite positions. A formation period of length l is considered in which the cumulative returns of the two portfolios are computed. The portfolio with the highest cumulative returns is the winner, the other portfolio is the loser. If, in the following h periods, the winner portfolio outperforms, in terms of cumulative returns, the loser portfolio, then momentum occurs. If the opposite is observed, then reversal occurs. Formally, define the cumulative return of the winner portfolio given a partial history σ_t,

$$W_{h,l}(\sigma_t) = \begin{cases} \sum_{i=0}^{h-1} (r(\sigma_{t-i}) - \beta^{-1}) & \text{if} \quad \sum_{j=h}^{h+l-1} (r(\sigma_{t-j}) - \beta^{-1}) > 0, \\ \sum_{i=0}^{h-1} (\beta^{-1} - r(\sigma_{t-i})) & \text{otherwise.} \end{cases}$$

Definition 3.3 (Cross-sectional momentum and reversal)**.** *Cross-sectional momentum* with a formation period of length l occurs h periods after formation if $\liminf_{t\to\infty} \mathrm{E}[W_{h,l}(\sigma_t)] > 0$. *Cross-sectional reversal* with a formation period of length l occurs h periods after formation if $\limsup_{t\to\infty} \mathrm{E}[W_{h,l}(\sigma_t)] < 0$.

If p is i.i.d. , then $\lim_{t\to\infty} \mathrm{E}[F_j(\sigma_t)] = \lim_{t\to\infty} \mathrm{Cov}[r(\sigma_{t+j}), r(\sigma_t)] = 0 \ \forall j$. Thus, neither conditional nor time-series momentum and reversal occur. With respect to the cross-sectional definitions,

$$\mathrm{E}[W_{1,1}(\sigma_{t+2})|\sigma_t] = \frac{-\pi^2 F_1(\sigma_t)}{\beta p(1|\sigma_t, 1) p(1|\sigma_t, 0)} + \frac{(1-2\pi)(p(1|\sigma_t, 0) - \pi)}{\beta p(1|\sigma_t, 0)}. \tag{2}$$

The second term on the right-hand side of (2) implies that the sign of the expected return of the winner portfolio is not exclusively decided by the occurrence of the time-series momentum. Thus, cross-sectional anomalies can be observed even with i.i.d. subjective measures.

More generally, if the belief of the agents is equal to or converges towards the true process almost surely, then the risk-neutral measure converges towards P and all anomalies disappear. Thus, in our framework, any pricing anomaly must emerge from a persistently incorrect way of assigning probabilities to states of nature. In particular, to prevent markets from being eventually efficient, agents should not be able to learn the truth.

4 Belief updating with investor sentiment

The agents in our model are characterized by three types of possible cognitive biases: *representativeness heuristic*, *conservatism*, and *adjustment and anchoring.* The representativeness heuristic induces people confronted with a random sequence of observations to believe that the essential characteristics of the data generation process will be represented even in short sequences, so that they tend to see some structure in random samples (Tversky and Kahneman, 1974). Conservatism indicates the tendency of people facing some piece of evidence to update subjective probabilities in the correct direction, but in an insufficient amount with respect to what the Bayes theorem would prescribe (Edwards, 1982). Finally, adjustment and anchoring refer to the observation that people tend to make sequential evaluations starting from an initial (anchoring) point and then adjusting. Thus, the results tend to be biased towards the initial values (Tversky and Kahneman, 1974).

In our model, we assume that agents try to learn the true process using two models. Those models are Markov chains described by the transition matrices

$$M_h : \begin{array}{c c} & \begin{array}{cc} s_{t+1}=0 & s_{t+1}=1 \end{array} \\ \begin{array}{c} s_t=0 \\ s_t=1 \end{array} & \begin{pmatrix} \pi_h & 1-\pi_h \\ 1-\pi_h & \pi_h \end{pmatrix} \end{array}, \quad h = 1, 2 \,. \tag{3}$$

Let $M_h(s_{t+1} \mid s_t)$ be the probability of observing state s_{t+1} after state s_t according to model $h = 1, 2$, then, $M_h(s_{t+1} = s_t \mid s_t) = \pi_h$ and $M_h(s_{t+1} \neq s_t \mid s_t) = 1 - \pi_h$. We set $\pi_1 < 0.5 < \pi_2$, so that M_1 assigns a higher probability to switch than to persist, while M_2 assigns a higher probability to remain in a state instead of switching. Given a partial history σ_t, the probability attached by each agent to the realization of s_{t+1} is a convex weighting of the probabilities assigned by two models,

$$p(s_{t+1}|\sigma_t) = \sum_{h=1}^{2} w_h(\sigma_t)\, M_h(s_{t+1}|s_t) \tag{4}$$

with $w_h(\sigma_t) \in [0, 1]$, $\sum_{h=1}^{2} w_h(\sigma_t) = 1$, and $p(\sigma_t) = \prod_{\tau=1}^{t} p(s_\tau|\sigma_{\tau-1})$. The weights are updated according to the following prescription,

$$w_1(\sigma_{t+1}) = \mu\lambda + (1-\mu)\frac{M_1(s_{t+1}|s_t)w_1(\sigma_t)}{p(s_{t+1}|\sigma_t)} \,, \tag{5}$$

with $w_2(\sigma_{t+1}) = 1 - w_1(\sigma_{t+1})$ and $\lambda, \mu \in [0, 1]$. The *representativeness heuristic* emerges when $\mu \in [0, 1)$. In this case, $w_1(\sigma_{t+1}) > w_1(\sigma_t)$ if $s_{t+1} \neq s_t$ and $w_1(\sigma_{t+1}) < w_1(\sigma_t)$ if $s_{t+1} = s_t$. Such a reinforcing mechanism, combined with the two Markov models, makes agents believe that the true process has positive autocorrelation if random sequences of equal states occur and that it has negative autocorrelation as random sequences of alternating states occur. The parameter λ describes *anchoring and adjusting.* Indicates the anchoring probability attached to M_1 while $1 - \lambda$ is that attached to M_2. The parameter μ expresses the degree of *conservatism.* If $\mu = 1$, the agents are maximally conservative without any representativeness heuristic. In this case, their beliefs are completely characterized by anchor probabilities. If $\mu = 0$, the agents are not affected by conservatism and behave as Bayesian. The intermediate values of μ indicate a belief adjustment process around the anchor with a certain degree of conservatism.

5 The emergence of pricing anomalies

Some sufficient or necessary conditions for the presence or absence of anomalies can be derived using the results about *reinforcing* and *progressive* update rules in Bottazzi and Giachini (2022). Reinforcing means that new evidence in favor of a model increases the weight assigned to it, while progressive means that the effect of a new realization is not reduced by previous evidence. Define the functions

$$f_{+/-}(w;\mu,\lambda) = \mu\lambda + (1-\mu)\frac{(1_+ - \pi_1)w}{1_+ - \pi_1 w - \pi_2(1-w)},$$

where 1_+ is equal to 1 for f_+ and 0 for f_-. Note that if $s_{t+1} \neq s_t$, $w_1(\sigma_{t+1}) = f_+(w_1(\sigma_t);\mu,\lambda)$, while if $s_{t+1} = s_t$, $w_1(\sigma_{t+1}) = f_-(w_1(\sigma_t);\mu,\lambda)$.

Proposition 5.1. *For any* $\lambda \in [0,1]$ *and* $\mu \in (0,1)$, *there exist* $\underline{w}_{\mu,\lambda}$ *and* $\overline{w}_{\mu,\lambda}$, *unique in* $(0,1)$, *that solve* $w = f_-(w;\mu,\lambda)$ *and* $w = f_+(w;\mu,\lambda)$, *respectively. It is* $\underline{w}_{\mu,\lambda} < \overline{w}_{\mu,\lambda}$ *and if* $w_1(\sigma_0) \in [\underline{w}_{\mu,\lambda}, \overline{w}_{\mu,\lambda}]$, *then* $w_1(\sigma_t) \in [\underline{w}_{\mu,\lambda}, \overline{w}_{\mu,\lambda}]$, $\forall \sigma_t$.

Moreover, the update rule in (5) *is progressive, that is,* f_- *and* f_+ *are continuous and nondecreasing in* $[\underline{w}_{\mu,\lambda}, \overline{w}_{\mu,\lambda}]$, *and reinforcing, that is,* $\forall w \in [\underline{w}_{\mu,\lambda}, \overline{w}_{\mu,\lambda}]$, $f_-(w) < w$ *and* $f_+(w) > w$.

Proof. See Appendix A. □

Figure 1 shows two examples of $f_+(w;\mu,\lambda)$ and $f_-(w;\mu,\lambda)$ with the relative $\underline{w}_{\mu,\lambda}$ and $\overline{w}_{\mu,\lambda}$. On the left, all the cognitive biases are active. On the right, the agents are Bayesians, $\underline{w}_{0,\lambda} = 0$, $\overline{w}_{0,\lambda} = 1$, and only the representative heuristics is active.

Proposition 5.2 (Conditional momentum and reversal)**.** *For any* $\lambda \in [0,1]$ *and* $\mu \in (0,1)$, *conditional momentum occurs if*

$$\mu\lambda + \underline{w}_{\mu,\lambda}\frac{(1-\pi_1)(1-\mu+\underline{w}_{\mu,\lambda}) + (1-\pi_2)(1-\underline{w}_{\mu,\lambda})}{1-\pi_1\underline{w}_{\mu,\lambda} - \pi_2(1-\underline{w}_{\mu,\lambda})} > \frac{2\pi_2 - 1}{\pi_2 - \pi_1}. \tag{6}$$

Conditional momentum does not occur if

$$\mu\lambda + \overline{w}_{\mu,\lambda}\frac{\pi_1(1-\mu+\overline{w}_{\mu,\lambda}) + \pi_2(1-\overline{w}_{\mu,\lambda})}{\pi_1\overline{w}_{\mu,\lambda} + \pi_2(1-\overline{w}_{\mu,\lambda})} < \frac{2\pi_2 - 1}{\pi_2 - \pi_1}. \tag{7}$$

Conditional reversal occurs if and only if

$$2\underline{w}_{\mu,\lambda} < \frac{2\pi_2 - 1}{\pi_2 - \pi_1}. \tag{8}$$

Proof. See Appendix B. □

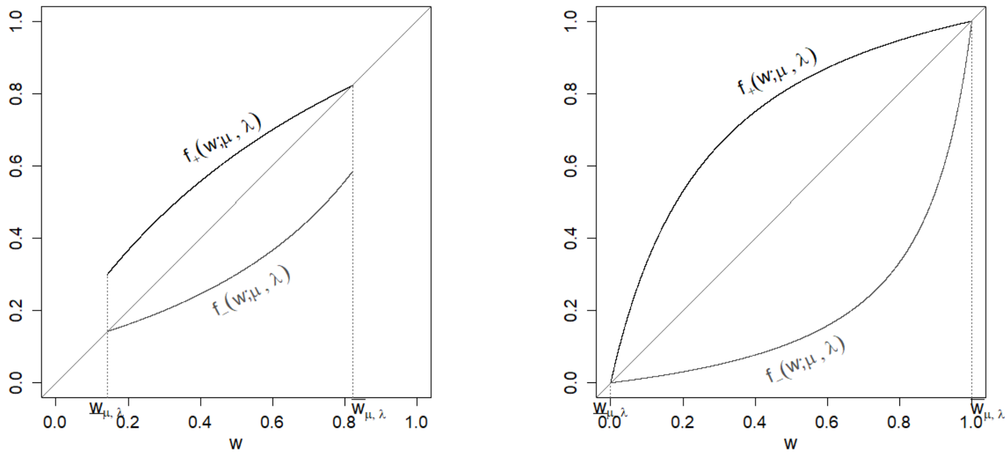

Figure 1: Examples of $f_+(w;\mu,\lambda)$ and $f_-(w;\mu,\lambda)$. Left: $\mu = 0.2$, $\lambda = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.6$. Right: $\mu = 0$, $\pi_1 = 0.1$, $\pi_2 = 0.8$.

Proposition 5.3 (Cross sectional momentum and reversal)**.** *For any $\lambda \in [0,1]$ and $\mu \in [0,1)$, cross sectional momentum with 1 formation period occurs 1 period after formation if (6) is satisfied and one of the following conditions also holds: $\pi = 1/2$, or $\pi < 1/2$ and $\underline{w}_{\mu,\lambda} \geq (\pi + \pi_2 - 1)/(\pi_2 - \pi_1)$, or $\pi > 1/2$ and $\overline{w}_{\mu,\lambda} \leq (\pi + \pi_2 - 1)/(\pi_2 - \pi_1)$.*

Cross sectional reversal with 1 formation period occurs 1 period after formation if (7) is satsified and one of following conditions also holds: $\pi = 1/2$, or $\pi < 1/2$ and $\overline{w}_{\mu,\lambda} \leq (\pi + \pi_2 - 1)/(\pi_2 - \pi_1)$, or $\pi > 1/2$ and $\underline{w}_{\mu,\lambda} \geq (\pi + \pi_2 - 1)/(\pi_2 - \pi_1)$.

Proof. See Appendix C. □

The conditions given above are not exhaustive on how cognitive biases lead to the emergence of pricing anomalies according to different definitions. In fact, they are completely silent on time series anomalies and more general cross-sectional effects. There are, however, two special cases that we can analyze in more depth.

5.1 The Bayesian and fully conservative cases

When $\mu = 0$, the agents are not affected by conservatism and adjust their beliefs in a Bayesian way. In this case, only the representativeness heuristic is present. Define the average entropy of the two Markov models with respect to the true i.i.d. process

$$D_P(M_h) = \pi^2 \log \frac{\pi}{\pi_h} + \pi(1-\pi) \log \frac{\pi(1-\pi)}{(1-\pi_h)^2} + (1-\pi)^2 \log \frac{1-\pi}{\pi_h}, \quad h = 1, 2.$$

A Bayesian learner converges to the Markov model with the lowest relative entropy (see Antico et al., 2023, for a similar model). If $D_P(M_1) < D_P(M_2)$, then P-almost surely $\lim_{t\to\infty} w_1(\sigma_t) = 1$, while if $D_P(M_1) > D_P(M_2)$, then P-almost surely $\lim_{t\to\infty} w_1(\sigma_t) = 0$.

In contrast, when $\mu = 1$, the agents show extreme conservatism, no representativeness heuristic, and no adjustment around the anchor. It is $w(\sigma_t) = \lambda$, $\forall \sigma_t$, and the subjective measure is a fixed convex combination of the two Markvov models.

In both previous cases, the subjective measure asymptotically converges towards a Markov process in which $\text{Prob}\{s_t = s_{t-1}\} = \pi_1 \tilde{w} + \pi_2(1 - \tilde{w}) = \tilde{\pi}$, with, $\tilde{w} = 0, 1, \lambda$ depending on the above conditions. In general,

$$\text{Cov}\left[r(\sigma_{t+2}), r(\sigma_{t+1})|\sigma_t\right] = -\frac{\pi(1-\pi)\,\text{E}\left[r(\sigma_{t+1})|\sigma_t\right] F_1(\sigma_t)}{\beta p(1|\sigma_t, 1) p(1|\sigma_t, 0)}.$$

If p is Markov, on almost all paths, $F_j(\sigma_t) = 2\tilde{\pi} - 1$, $\forall j$, so that the conditional and time-series definitions of momentum coincide, and $\text{Cov}[r(\sigma_{t+j}), r(\sigma_t)] = 0$, $\forall j > 1$. These observations directly imply the following.

Proposition 5.4 (Conditional and time-series anomalies)**.** *Conditional momentum and time-series momentum occur, while conditional reversal and time-series reversal do not occur, if*

$$2\tilde{w} > \frac{2\pi_2 - 1}{\pi_2 - \pi_1}.$$

Conditional reversal occurs, while time-series reversal, conditional momentum, and time-series momentum do not occur, if

$$2\tilde{w} < \frac{2\pi_2 - 1}{\pi_2 - \pi_1}.$$

With respect to cross-sectional anomalies, the following result can be proved.

Proposition 5.5 (Cross-sectional anomalies)**.** *There exists a number $\phi \in [0.5, 1)$ that depends on the true probability π such that cross-sectional momentum with the formation period of* 1 *occurs* 1 *period after formation if $\tilde{w} > (\pi_2 - \phi)/(\pi_2 - \pi_1)$, while cross-sectional reversal with the formation period of* 1 *occurs* 1 *period after formation if $\tilde{w} < (\pi_2 - \phi)/(\pi_2 - \pi_1)$.*

Proof. See Appendix D. □

In conclusion, representativeness heuristic or full conservatism alone is enough to observe a wide range of pricing anomalies. However, it is generally not possible to relate one of these anomalies to a specific bias.

6 Numerical exploration

We present a series of numerical exercises that explore the occurrence of pricing anomalies for generic values of $\mu \in (0,1)$ and $\lambda \in [0,1]$. Starting with conditional and time-series anomalies, we consider a sufficiently large time horizon T, choose a s_0, and draw M sequences of realizations $\sigma_{m,T} = (s_0, s_{m,1}, s_{m,2}, \ldots, s_{m,T})$ with $m = 1, 2, \ldots, M$. Then, we average over the replica and compute

$$\widehat{F}_1 = \frac{1}{M} \left(\sum_{m=1}^{M} p(1|\sigma_{m,T}, 1) - p(1|\sigma_{m,T}, 0) \right)$$

and

$$\widehat{\rho}(j) = \frac{\left(\sum_{m=1}^{M} \left(r(\sigma_{m,T}) - \overline{r}_T\right) \left(r(\sigma_{m,T-j}) - \overline{r}_{T-j}\right) \right)}{\sqrt{\sum_{m=1}^{M} \left(r(\sigma_{m,T}) - \overline{r}_T\right)^2} \sqrt{\sum_{m=1}^{M} \left(r(\sigma_{m,T-j}) - \overline{r}_{T-j}\right)^2}},$$

where $\overline{r}_t = M^{-1} \sum_{m=1}^{M} r(\sigma_{m,t})$. Using correlations instead of covariances is statistically convenient, as we can use the approximation to the Student's t distribution in the null case to evaluate significance. $T = 100$ is sufficient to obtain reliable results that do not change with longer simulations. We set $M = 100000$ so that the error bands at the $\sim 97.5\%$ confidence level are negligible given the size of the picture.

A significantly negative value of $\widehat{F}_1$ is evidence of conditional momentum, while a significantly positive value of $\widehat{\rho}(1)$ is evidence of time-series momentum. Figure 2 shows the combinations of (π_1, π_2) for which momentum is observed. The solid black line delimits the area where the sufficient condition for the conditional momentum in (6) is satisfied. A widespread concordance of the two definitions is apparent, even if there exist regions in which they provide opposite results. The two definitions seem to strongly agree when conservatism is high, whereas disagreement is observed in some regions of the parameter space when conservatism is low. Note that with high conservatism the theoretically sufficient condition for conditional momentum appears (almost) necessary. This is due to the similarity of the subjective measure to a Markov process. The anchoring parameter λ decides the extent of the region where the effects are observed. When λ is large and anchoring favors Model 1, conditional and time series momentum occur in a wide region of the (π_1, π_2) plane. When, instead, λ is small and anchoring favors Model 2, the region where the effects are observed shrinks significantly. This is expected as momentum is mainly driven by Model 1.

A similar investigation of the occurrence of reversal is reported in Figure 3. For conditional reversal, we simply use the theoretical condition derived in Proposition

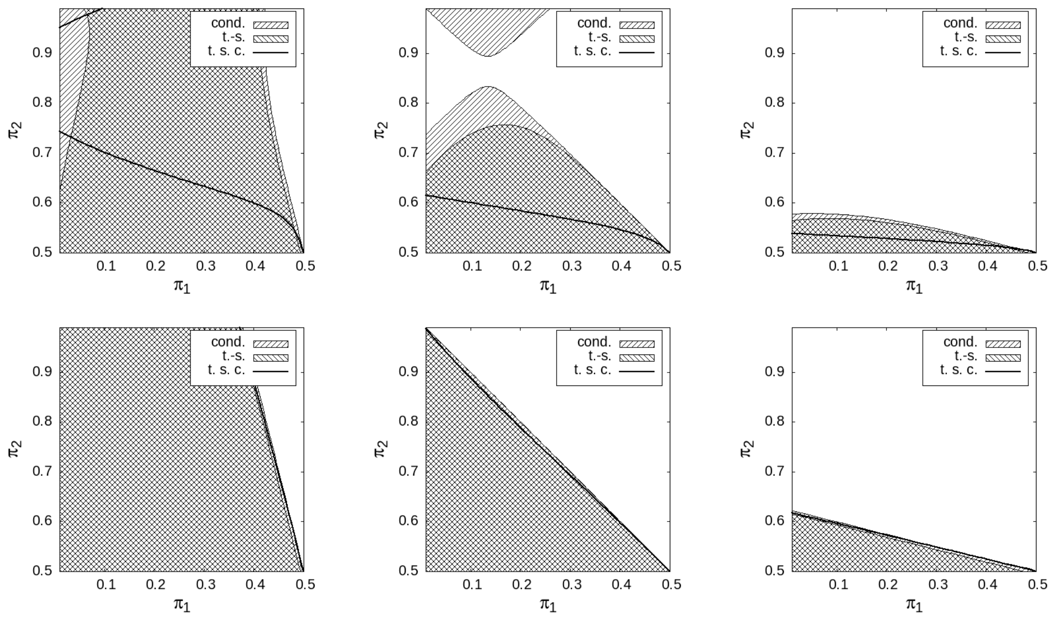

Figure 2: Occurrence of conditional momentum (cond.), time-series momentum (t.-s.), and the theoretical sufficient condition for conditional momentum (t.s.c.) for different combinations of (π_1, π_2), $\pi = 0.5$, and representative values of μ and λ. In the upper panels, conservatism is low ($\mu = 0.2$), in the lower panels, conservatism is high ($\mu = 0.8$). In the left panels, the anchoring favors Model 1 ($\lambda = 0.8$), in the center panels the anchoring is symmetric ($\lambda = 0.5$), and in the right panels the anchoring favors Model 2 ($\lambda = 0.2$).

5.3. For time-series reversal, we check whether for at least one lag $j \in \{2, 3, \ldots, 15\}$ it is $\widehat{\rho}(j)$ significantly negative with confidence level 97.5%. Iime-series reversal is observed for almost all (π_1, π_2) except for a small region close to $(0.5, 0.5)$. There, the agents converge to having correct expectations and the autocorrelations tend to zero. Conservatism and anchoring appear to have a limited effect on time-series reversal. In contrast, conditional reversal is strongly influenced by behavioral parameters. It is present for almost any combination of π_1 and π_2 if conservatism is low and anchoring favors Model 2. As conservatism rises or anchoring starts favoring Model 1, the region in which it occurs shrinks. This is expected, as the conditional reversal is driven primarily by Model 2. Thus, as anchoring favors it or the learning rule can converge to it (as in the Bayesian limit), such an anomaly becomes increasingly widespread. The conditional and time-series definitions of reversal tend to agree when both π_1 and π_2 are close to their upper limits, while they tend to disagree when the two parameters are close to their lower limits.

Figure 4 shows the regions where cross-sectional anomalies occur one period

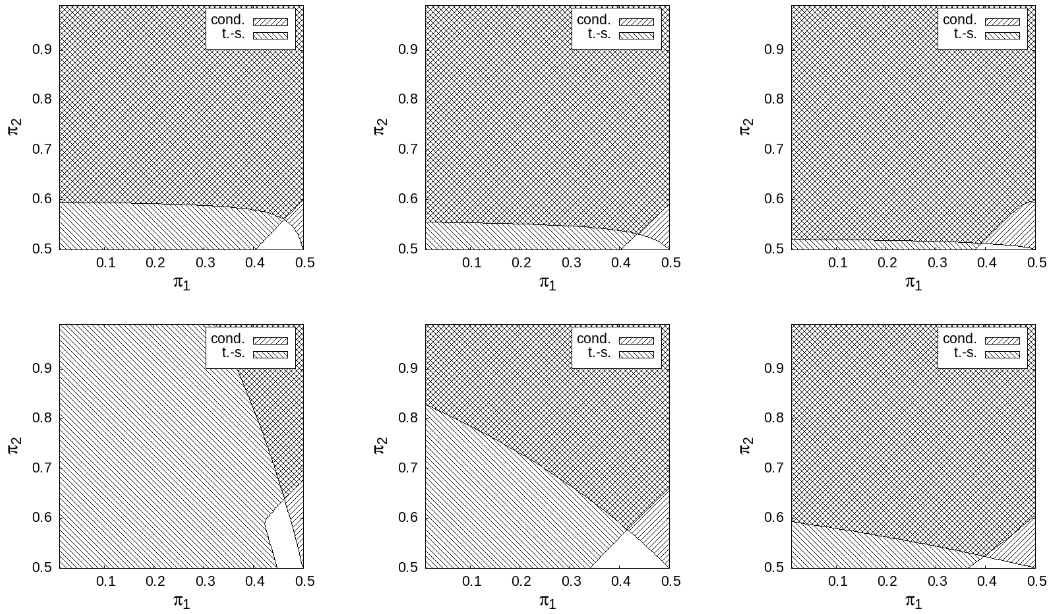

Figure 3: Occurrence of conditional reversal (cond.) and time-series reversal (t.-s.) for different combinations of (π_1, π_2), $\pi = 0.5$, and selected values of μ and λ. In the upper panels, conservatism is low ($\mu = 0.2$), in the lower panels, conservatism is high ($\mu = 0.8$). In the left panels, the anchoring favors Model 1 ($\lambda = 0.8$), in the center panels the anchoring is symmetric ($\lambda = 0.5$), and in the right panels the anchoring favors Model 2 ($\lambda = 0.2$).

after formation for portoflios with one period formation, considering the same parameter values used in the previous exercises. Cross-sectional momentum is observed when π_2 is close to 0.5, while cross-sectional reversal tends to be observed when π_1 is close to 0.5. As agents become more conservative, the conditions appear to be tighter and largely overlap with the conditions for conditional momentum and conditional reversal. However, we know that when $\pi = 0.5$, the sufficient condition for cross-sectional momentum reduces to the sufficient condition for conditional momentum. Therefore, in Figure 5, we repeat the exercise considering $\pi = 0.25$. In this case, the regions where sufficient conditions are satisfied are reduced by the stricter requirements of Proposition 5.3. Cross-sectional reversal is the most affected and now tends to occur when $(\pi_1, `\pi_2) \sim (0.5, 1)$. It disappears when conservatism is high and anchoring favors Model 1.

Next, we investigate the performance of the Winner and Loser portfolio at different horizons and with different formation periods. Following Bottazzi et al. (2019), we use a computational adaptation of the estimation strategy commonly used in empirical contributions. That is, we draw a time series of T steps and

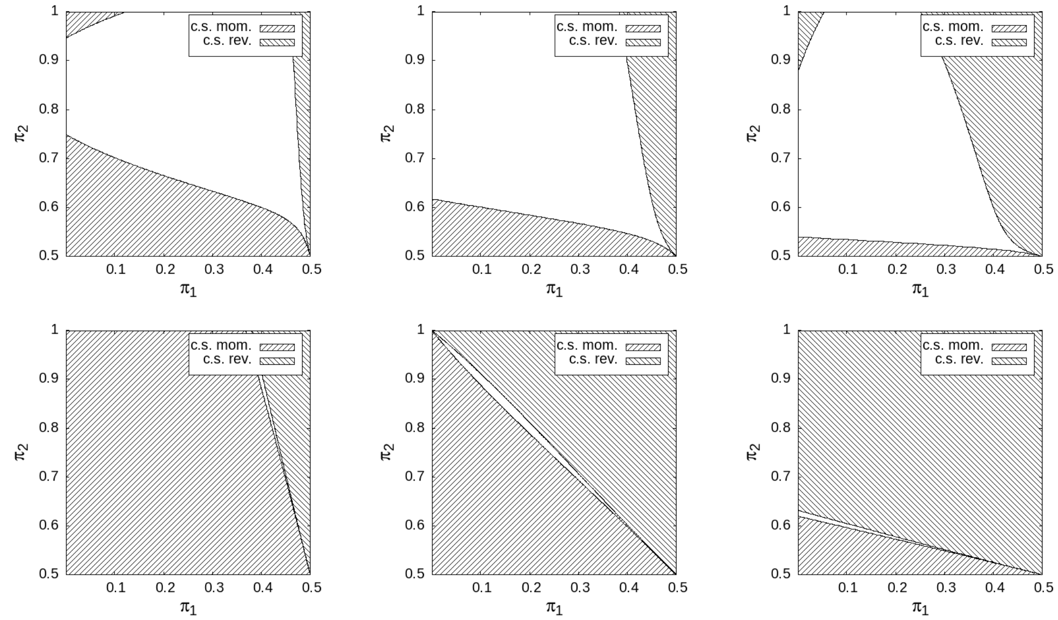

Figure 4: Occurrence of cross-sectional momentum (c.s. mom.) and reversal (c.s. rev.) for portfolios with formation period of one, one period after formation, for different combinations of (π_1, π_2), $\pi = 0.5$, and selected values of μ and λ. In the upper panels, conservatism is low ($\mu = 0.2$), in the lower panels, conservatism is high ($\mu = 0.8$). In the left panels, anchoring favors Model 1 ($\lambda = 0.8$), in the center panels, anchoring is symmetric ($\lambda = 0.5$), and in the right panels anchoring favors Model 2 ($\lambda = 0.2$).

divide it into L non-overlapping time windows of equal length l.[2] Winner and Loser portfolios are created at the end of each time window as described in Section 3 and their cumulative return is computed for each time step $h = 1, 2 \ldots, H$ following portfolio formation. Then, for each performance period h, cumulative returns are averaged over the L different performance windows after each portfolio formation, to obtain an estimate of the expected cumulative return h periods after formation.[3] Figure 6 shows the performance of the two portfolios for selected parameters values and different durations of the formation period. The occurrence of cross-sectional momentum is a rather recurrent feature. We find evidence of it for at least one horizon in any case considered. This is consistent with what is observed in Figure 2. The situation is the opposite for reversal. We have evidence of it only in the case of low conservatism and a formation period of length 6, while Figure 3 shows that

[2] This is the approach followed by De Bondt and Thaler (1985, 1987). Jegadeesh and Titman (1993, 2001) allow for overlapping windows in order to increase the sample size.

[3] Standard errors and confidence bands are computes as in De Bondt and Thaler (1985)

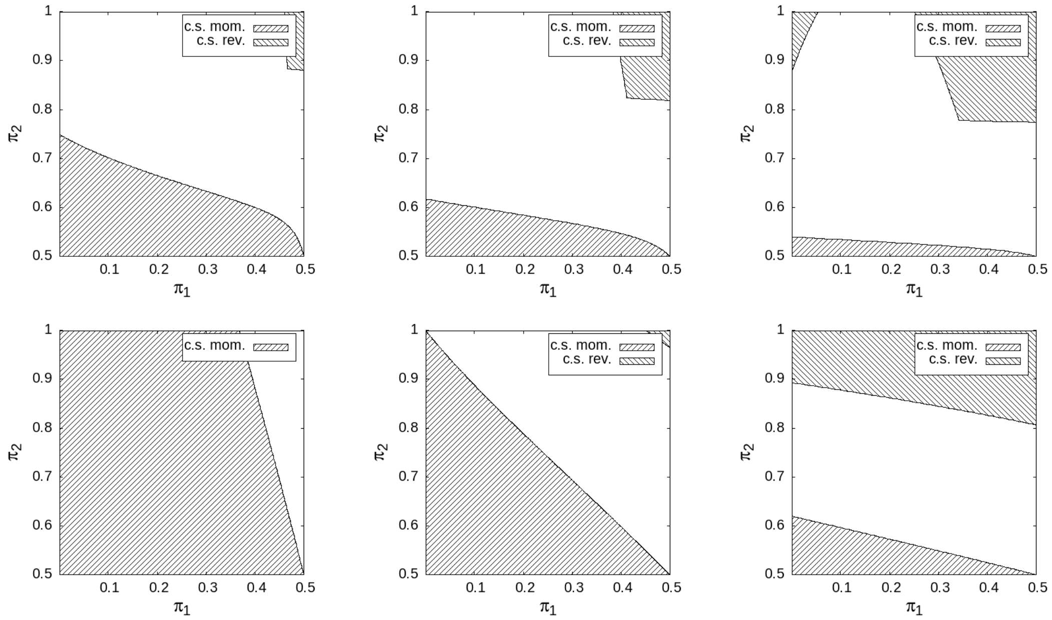

Figure 5: Occurrence of cross-sectional momentum (c.s. mom.) and reversal (c.s. rev.) for portfolios with formation period of one, one period after formation, for different combinations of (π_1, π_2), $\pi = 0.25$, and selected values of μ and λ. In the upper panels, conservatism is low ($\mu = 0.2$), in the lower panels, conservatism is high ($\mu = 0.8$). In the left panels, the anchoring favors Model 1 ($\lambda = 0.8$), in the center panels, the anchoring is symmetric ($\lambda = 0.5$), and in the right panels anchoring favors Model 2 ($\lambda = 0.2$).

time-series reversal always occurs and conditional reversal occurs if conservatism is low. Regarding the effect of the formation period, with $l = 1$ the Winner portfolio shows the highest performance in the first period and then stabilizes around a given level, for longer formation periods it tends to show an increasing behavior.

The predominance of the cross-sectional momentum in Figure 6 can be driven by the anchoring that favors Model 1. Therefore, in Figure 7 we repeat the exercises, selecting a value for the anchoring parameter that favors Model 2. As expected, now cross-sectional reversal emerges for any horizon considered if the formation period has length 1. Notice that the lack of momentum and the presence of a reversal effect are consistent with the indications provided by the conditional and time-series definitions. However, as the formation period increases, the Winner portfolio improves its performance and cross-sectional momentum is observed at any horizon considered if $l = 24$. Thus, increasing the formation period one passes from a situation in which there is perfect concordance among definitions to a situation in which cross-sectional anomalies provide opposite indications than

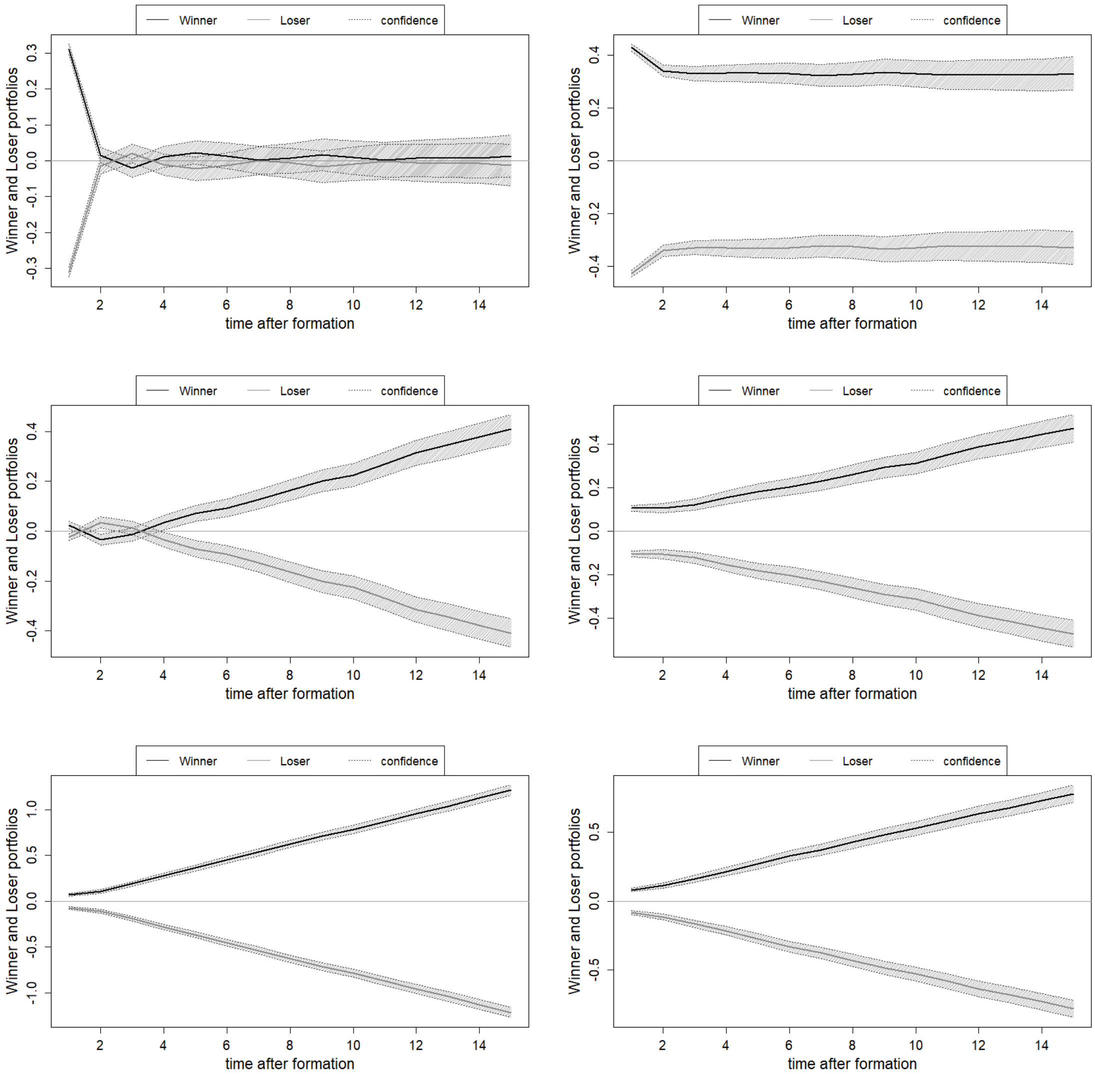

Figure 6: Winner and Loser portfolios' average cumulative returns with $H = 15$, $L = 100000$, $\pi = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.8$, $\lambda = 0.8$, $\mu \in \{0.2, 0.8\}$, and different formation periods. In the upper panels $l = 1$, in the center panels $l = 6$, and in the lower panels $l = 24$. In the left panels, conservatism is low ($\mu = 0.2$), in the right panels, conservatism is high ($\mu = 0.8$). The confidence bands are 3 standard errors away from estimates.

conditional and time-series ones.

A final point that we investigate is whether our model can produce cross-sectional momentum for short horizons followed by cross-sectional reversal for longer horizons, consistent with the empirical evidence of short-term momentum

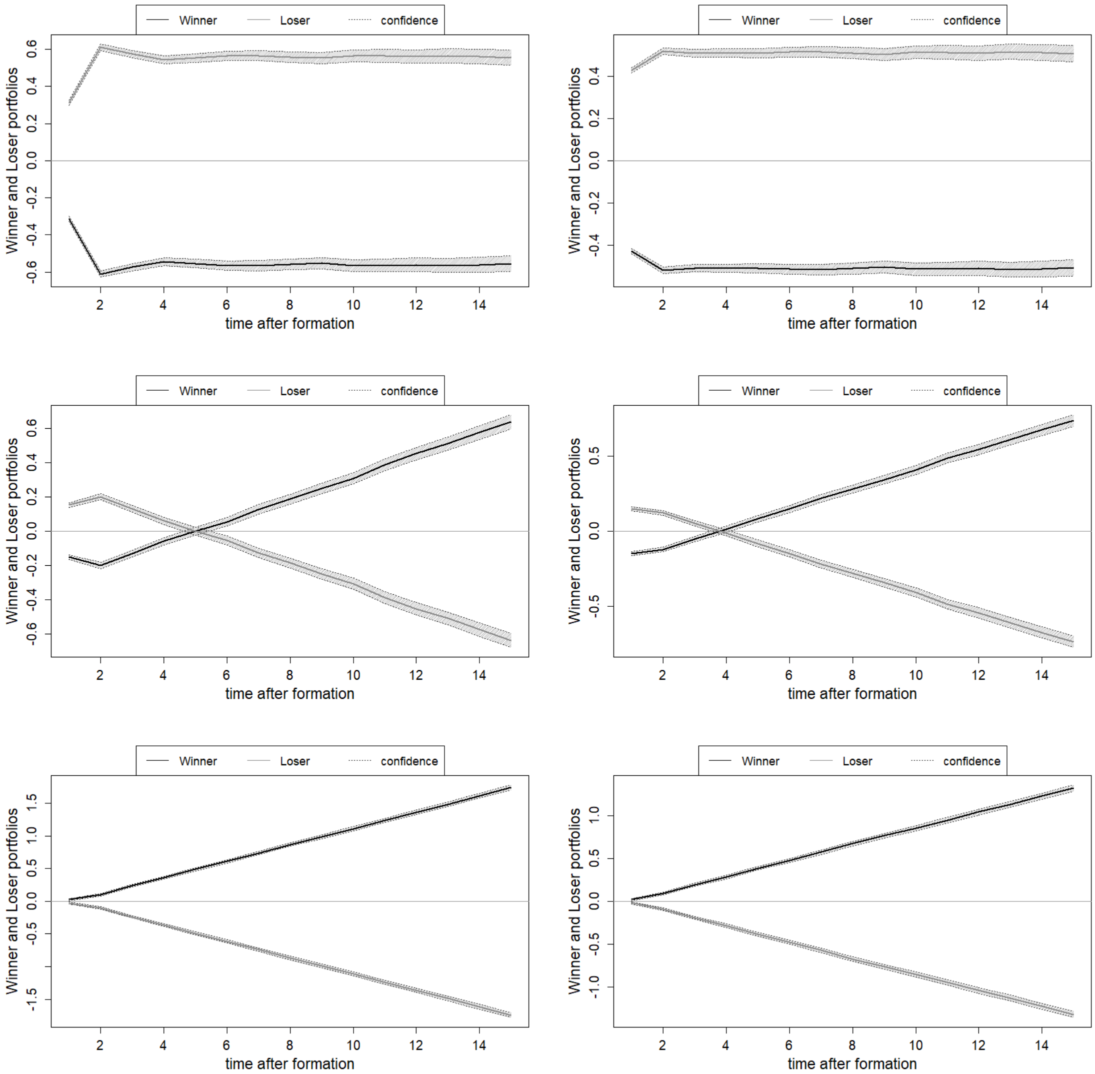

Figure 7: Winner and Loser portfolios' average cumulative returns with $H = 15$, $L = 100000$, $\pi = 0.5$, $\pi_1 = 0.2$, $\pi_2 = 0.8$, $\lambda = 0.2$, $\mu \in \{0.2, 0.8\}$, and different formation periods. In the upper panels $l = 1$, in the center panels $l = 6$, in the lower panels $l = 24$. In the left panels, conservatism is low ($\mu = 0.2$), in the right panels conservatism is high ($\mu = 0.8$). The confidence bands are 3 standard errors away from estimates.

and long-term reversal reported in the literature. The answer is positive and Figure 8 shows an example of such an occurrence. However, according to Proposition 5.3, conditional reversal does not occur for these parameter values. At the same time, conditional and time-series momentum and reversal are observed for combi-

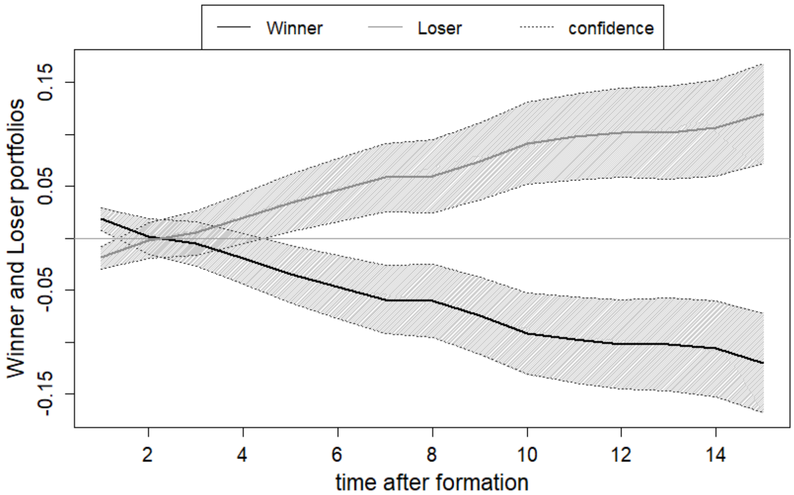

Figure 8: Winner and Loser portfolios' average cumulative returns with $H = 15$, $L = 100000$, $l = 2$, $\pi = 0.5$, $\pi_1 = 0.31$, $\pi_2 = 0.51$, $\lambda = 0.4$, and $\mu = 0.5$. Confidence bands are 3 standard errors away from estimates.

nations of parameter values for which cross-sectional anomalies are not observed (e.g., the top-left panel of Figure 6). Thus, disagreement among the definitions also emerges in terms of which combination of parameters shows momentum profits that subsequently revert. We must note here that extensive numerical explorations have not provided evidence of regions of the parameter space where all the anomalies occur together.

7 Conclusions

Using a simple and streamlined general equilibrium model, we study the emergence of momentum and reversal as a consequence of cognitive biases in learning. Following the different approaches existing in the literature, three definitions of the momentum and reversal pricing anomalies are considered: conditional, time-series, and cross-sectional. To explore how the different definitions interact and connect with specific behavioral characteristics, we consider a biased learning process that accounts for *representativeness heuristic*, *conservatism*, and *anchoring and adjusting*. The weight update rule is equivalent to that proposed in Antico et al. (2023) and represents an adaptation of the rule in Barberis et al. (1998) to a general equilibrium framework. This framework allows us to overcome the limits imposed by the single-security partial equilibrium setting that characterizes most of the theoretical literature on momentum and reversal and extend our analysis to the cross-sectional definitions often adopted in the empirical literature.

Our study shows that, despite the simplicity of the model, the three definitions, even if related, can generically differ and provide opposite indications on the occurrence of pricing anomalies. This suggests that they plausibly capture different aspects of mispricing, and only their joint study allows for a more complete picture of the behavioral causes of pricing anomalies and a better match of empirical evidence. At the behavioral level, an important finding is that conservatism and representative heuristics are not explicitly connected to the emergence of one of the anomalies. Additionally, some pricing anomalies are more affected by behavioral biases than others. For instance, while for other anomalies anchoring plays a role in determining how widespread they are, time-series reversal appears quite insensible to it. The analysis of cross-sectional effects shows that the length of the formation period and the horizon at which the effects can be observed add another layer of complexity and potential disagreement with the other definitions. Overall, the model can reproduce the empirical evidence of momentum profits that subsequently revert, even if the combinations of parameters change according to the adopted definitions.

The proposed model is highly stylized and presents several limitations. For instance, our numerical exploration has found no combination of the parameter values such that the pattern of momentum profits that subsequently revert is observed according to all definitions. Following Milian (2015), this is probably related to the assumption of homogeneous agents. Hence, one can imagine that adding agents with different belief formation rules to our model may let it match the empirical evidence more closely. At the same time, considering heterogeneous agents makes pricing more complicated and generates nontrivial market selection dynamics. This motivates future contributions aimed at carefully investigating the interaction between heterogeneous agents, pricing anomalies, and selection.

A Proof of Proposition 5.1

By direct inspection, it is immediately verified that in $[0,1]$, f_+ and f_- are continuous and increasing, f_+ is strictly concave and f_- is strictly convex, $f_+(0) = f_-(0) = \mu\lambda$ and $f_+(1) = f_-(1) = 1 - \mu(1-\lambda)$. These observations are sufficient to prove the statements of the theorem. The solutions of the equations read

$$\underline{w}_{\mu,\lambda} = \frac{1}{2} + \frac{\lambda\pi_2 + (1-\lambda)\pi_1}{2(\pi_2 - \pi_1)}\mu - \sqrt{\left(\frac{1}{2} + \frac{\lambda\pi_2 + (1-\lambda)\pi_1}{2(\pi_2 - \pi_1)}\mu\right)^2 - \frac{\mu\lambda\pi_2}{\pi_2 - \pi_1}},$$

and

$$\overline{w}_{\mu,\lambda} = \frac{1}{2} - \frac{1-\lambda\pi_2-(1-\lambda)\pi_1}{2(\pi_2-\pi_1)}\mu + \sqrt{\left(\frac{1}{2} - \frac{1-\lambda\pi_2-(1-\lambda)\pi_1}{2(\pi_2-\pi_1)}\mu\right)^2 + \frac{\mu\lambda(1-\pi_2)}{\pi_2-\pi_1}}.$$

B Proof of proposition 5.2

Given that the learning rule, for the parameters considered, is progressive and reinforcing (see Theorem 5.1), the first statement is derived from Proposition 4.1 and the second from Proposition 4.2 in Bottazzi and Giachini (2022).

C Proof of proposition 5.3

If (6) is respected, then $F_1(\sigma_t) < 0$. Thus, from (2), $\mathrm{E}[W_{1,1}(\sigma_{t+2})|\sigma_t] > 0$ if

$$(1-2\pi)(p(1|\sigma_t, 0) - \pi) = (1-2\pi)\left(w(\sigma_t, 0)(\pi_2 - \pi_1) + 1 - \pi_2 - \pi\right) \geq 0.$$

Recalling that $\underline{w}_{\mu,\lambda} \leq w(\sigma_t, 0) \leq \overline{w}_{\mu,\lambda}$, the first set of sufficient conditions is simply derived. For the second set, note that if (7) is respected, then $F_1(\sigma_t) > 0$. Thus, from (2), $\mathrm{E}[W_{1,1}(\sigma_{t+2})|\sigma_t] < 0$ if the previous inequality is satisfied in the opposite direction.

D Proof of Proposition 5.5

Because the subjective measure is Markov, from (2), it is

$$\lim_{t\to\infty} \mathrm{E}[W_{1,1}(\sigma_t)] = \frac{\pi^2}{\beta}\left(\frac{1}{\tilde{\pi}} - \frac{1}{1-\tilde{\pi}}\right) + \frac{(1-2\pi)(1-\tilde{\pi}-\pi)}{\beta(1-\tilde{\pi})} = \frac{\tilde{\pi}^2(2\pi-1) + \tilde{\pi}(1-3\pi) + \pi^2}{\beta\tilde{\pi}(1-\tilde{\pi})}.$$

The sign of the above expression is decided by the sign of the second-order polynomial in the numerator. This polynomial is equal to π^2 when $\tilde{\pi} = 0$, to $1/4 - \pi(1-\pi)$ when $\tilde{\pi} = 0.5$ and to $-\pi(1-\pi)$ when $\tilde{\pi} = 1$. Thus, there exists a single number $\phi \in [0.5, 1)$ such that if $\tilde{\pi} < \phi$ the expectation is positive, while if $\tilde{\pi} > \phi$ it is negative. The statement is recovered from the definition of $\tilde{\pi}$. If $\pi = 1/2$, $\phi = 1/2$, while if $\pi \neq 1/2$,

$$\phi = \frac{3\pi - 1 - \sqrt{1 - 6\pi + 13\pi^2 - 8\pi^3}}{4\pi - 2}.$$

Acknowledgements

The authors would like to thank Remco Zwinkels and the participants to the 4th YETI meeting for their comments.

Declaration of interest: none.

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